

# Model-based Control of Soft Robots: From First Principles to Learned Models

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with work performed by members of the RAD lab

Aug 7, 2020

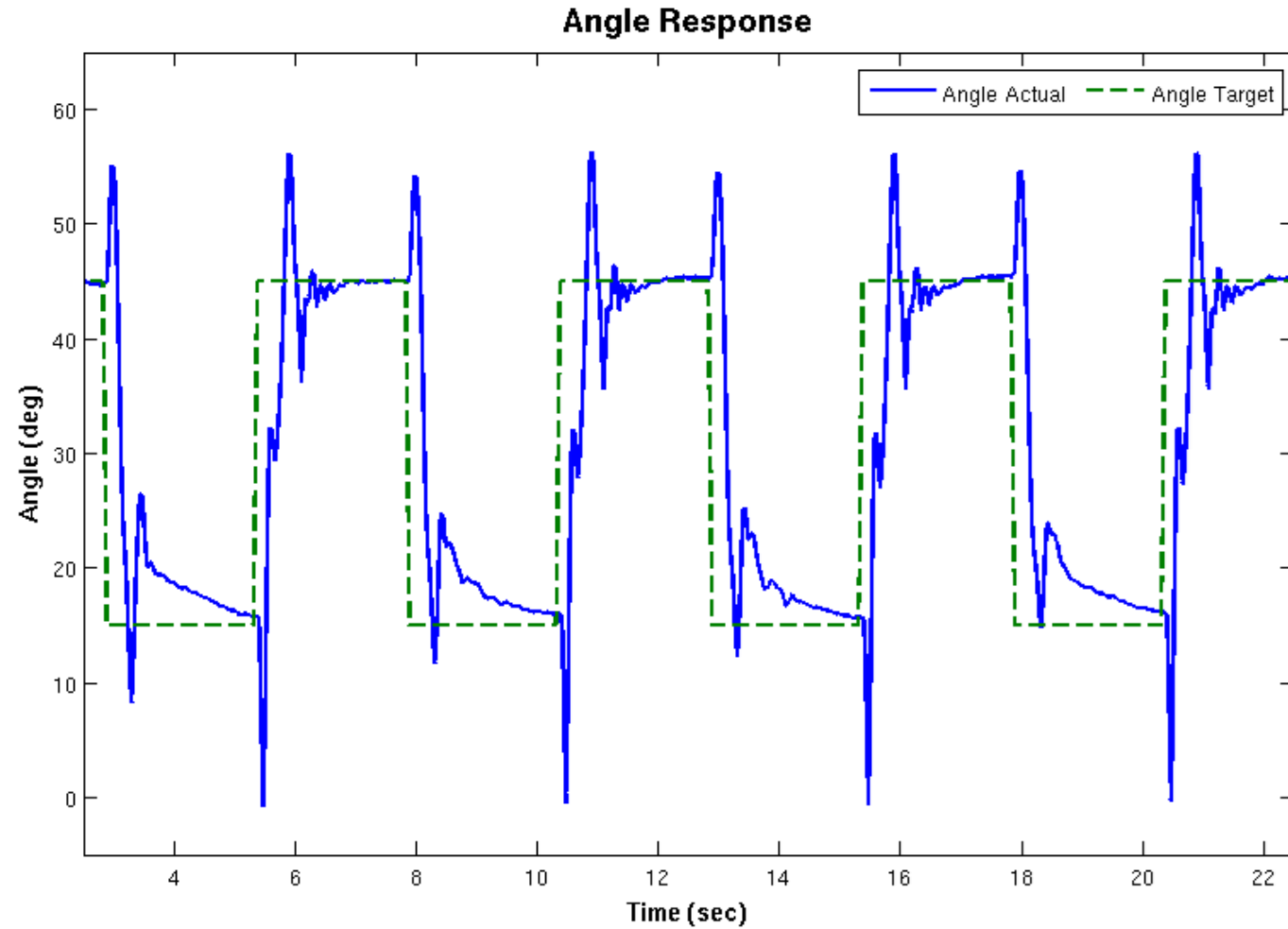
IEEE Robosoft Workshop on Modeling Soft Robots



# Acknowledgements

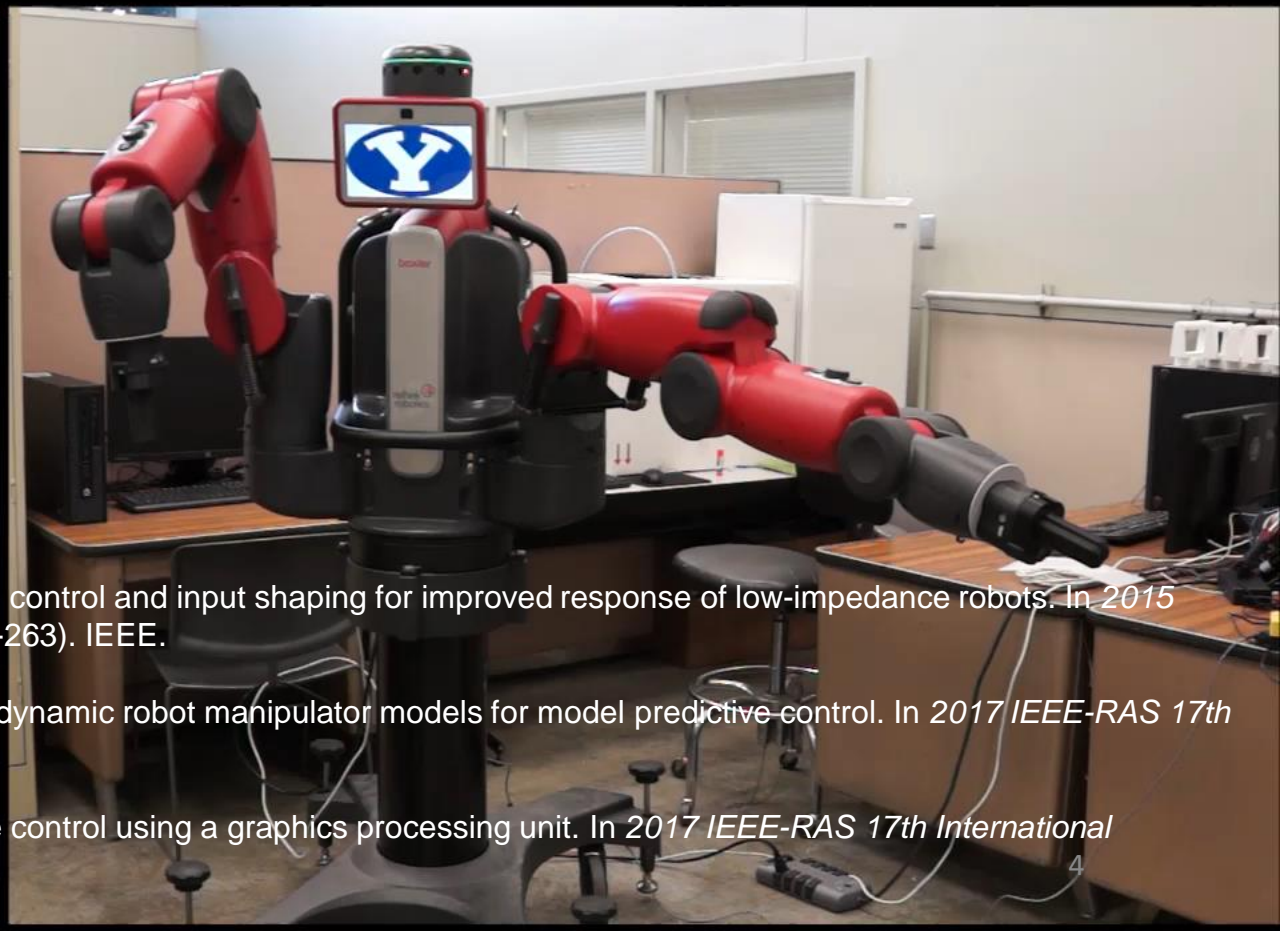
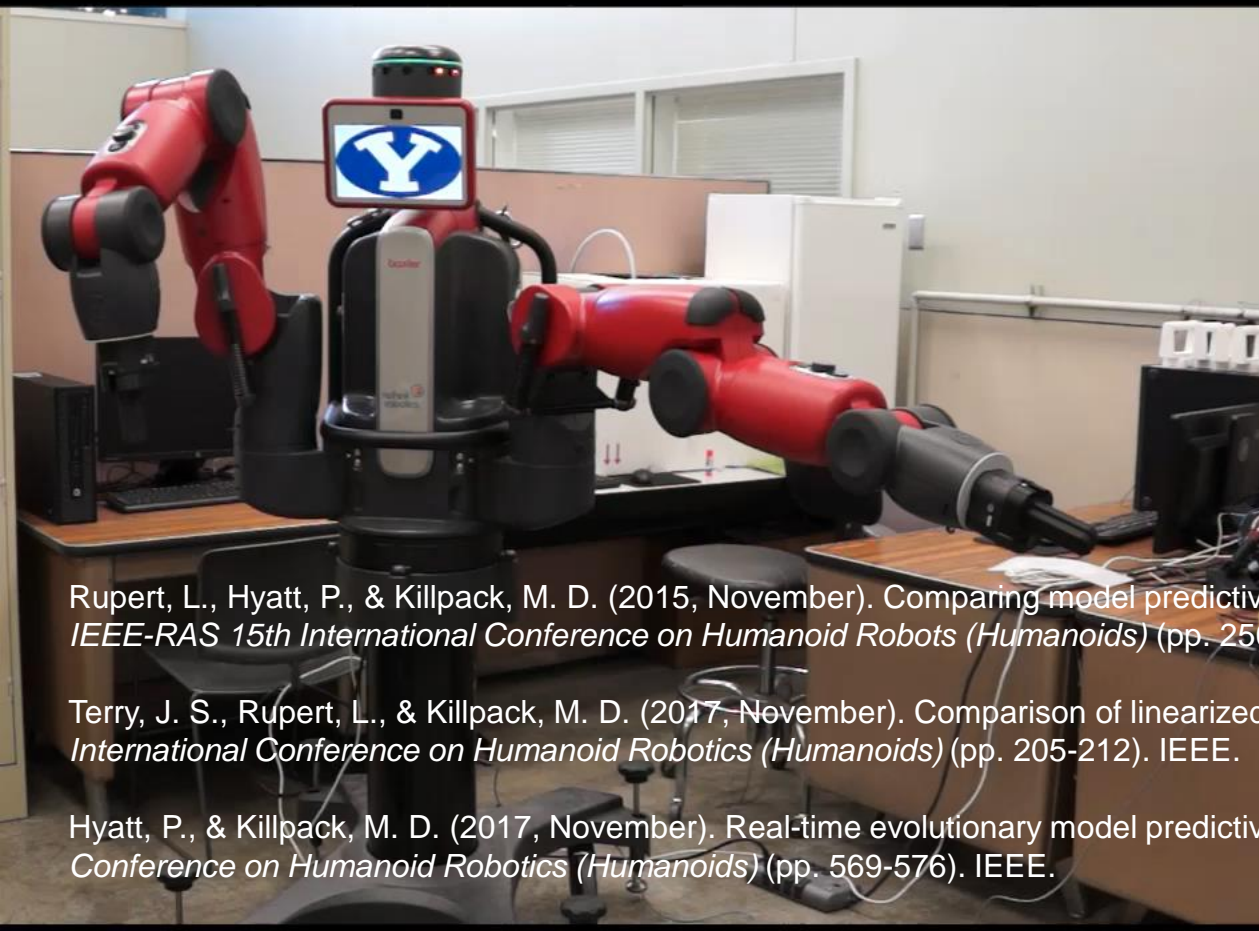


# Why Model-based Control?



# Impedance Control

# Model Predictive Control



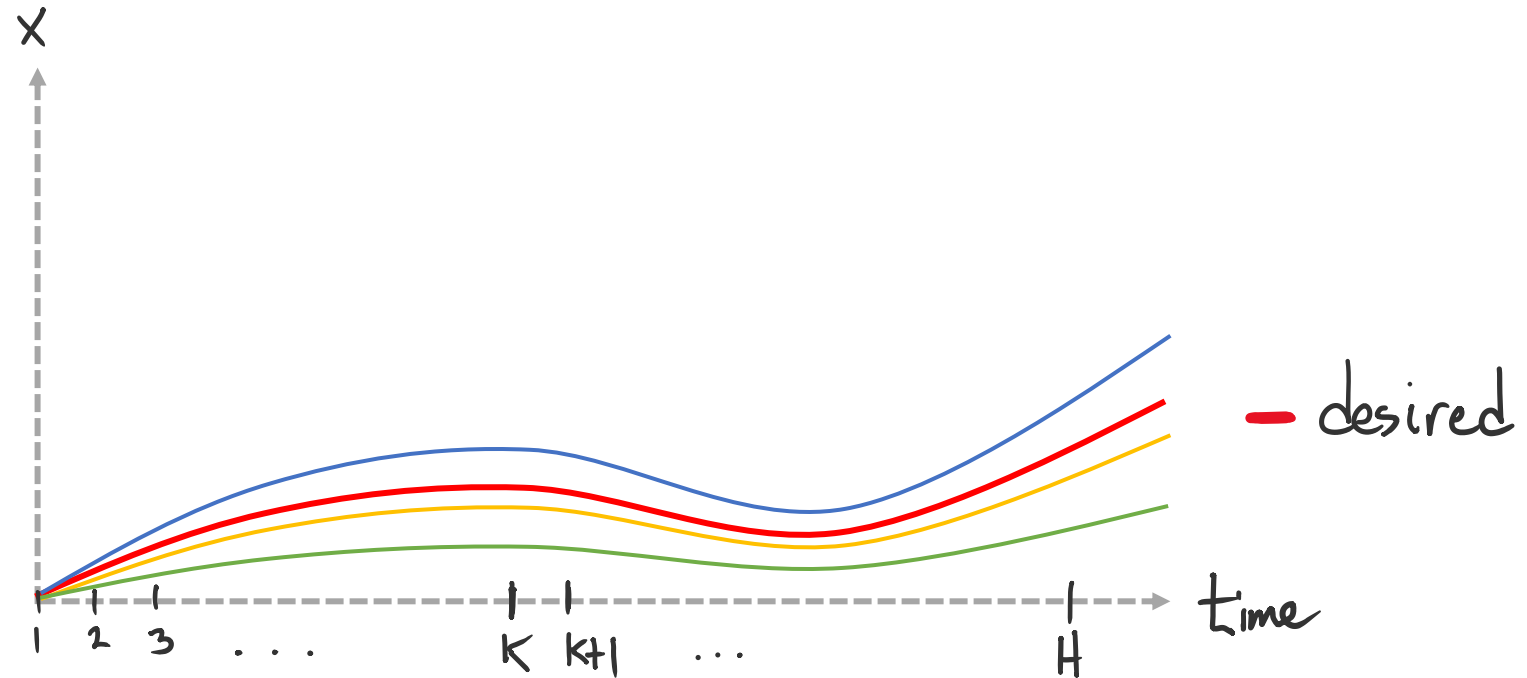
Rupert, L., Hyatt, P., & Killpack, M. D. (2015, November). Comparing model predictive control and input shaping for improved response of low-impedance robots. In *2015 IEEE-RAS 15th International Conference on Humanoid Robots (Humanoids)* (pp. 256-263). IEEE.

Terry, J. S., Rupert, L., & Killpack, M. D. (2017, November). Comparison of linearized dynamic robot manipulator models for model predictive control. In *2017 IEEE-RAS 17th International Conference on Humanoid Robotics (Humanoids)* (pp. 205-212). IEEE.

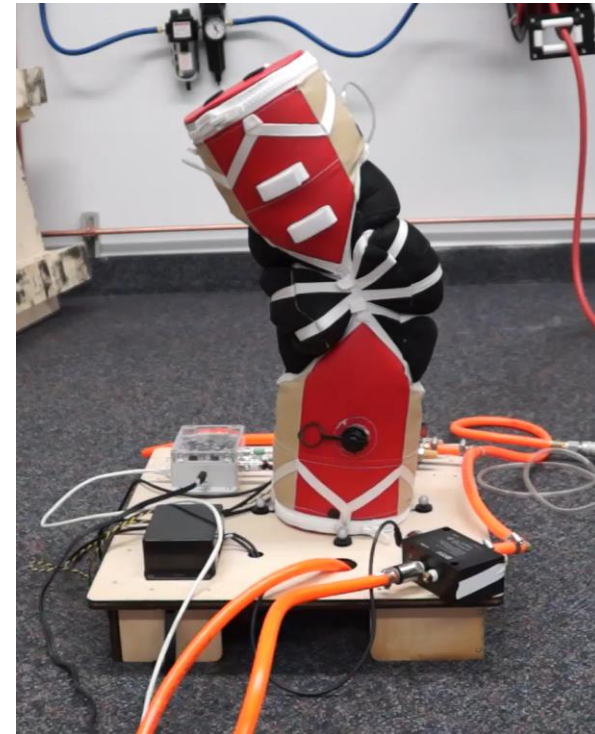
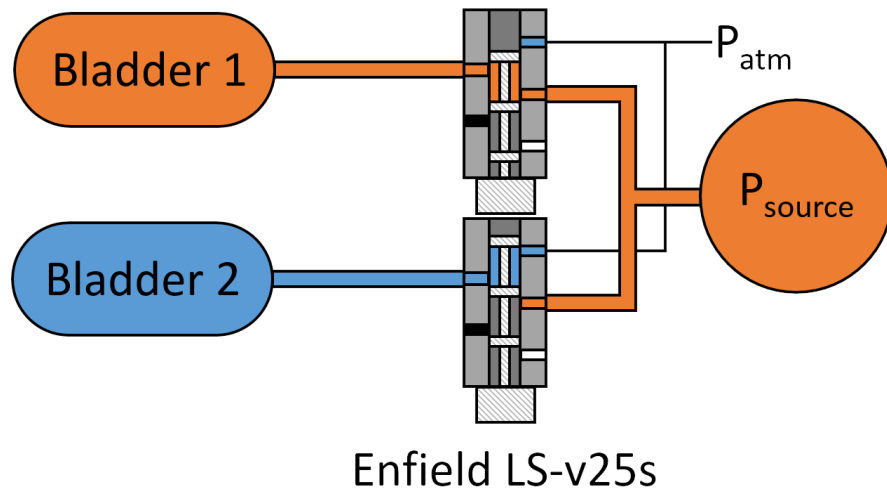
Hyatt, P., & Killpack, M. D. (2017, November). Real-time evolutionary model predictive control using a graphics processing unit. In *2017 IEEE-RAS 17th International Conference on Humanoid Robotics (Humanoids)* (pp. 569-576). IEEE.

# How Is the Dynamics Model Used?

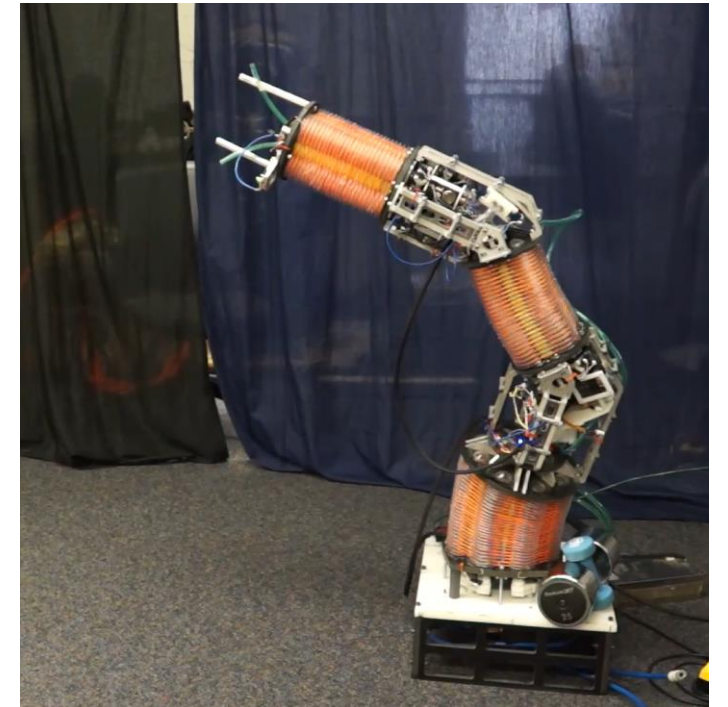
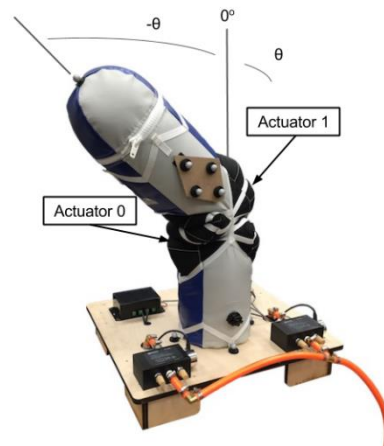
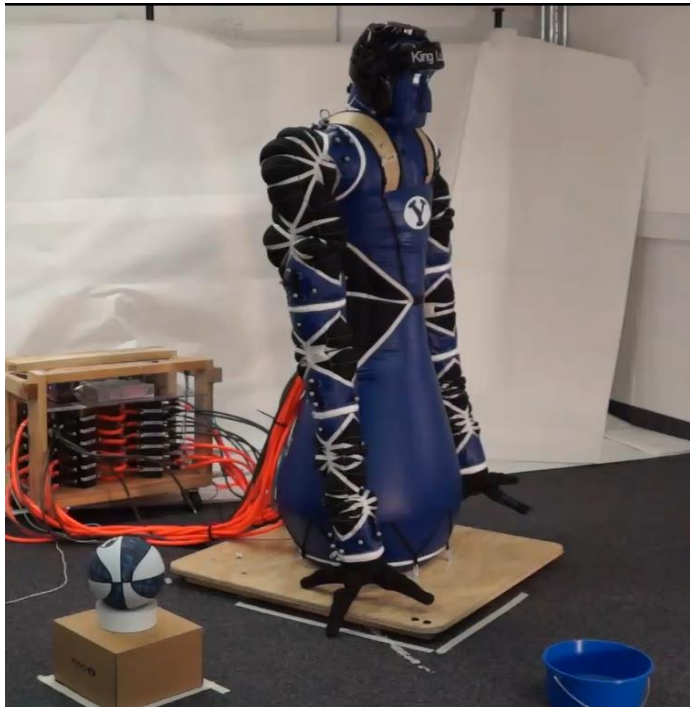
$$\dot{x} = f(x, u) \quad \longrightarrow \quad x_{k+1} = f(x_k, u_k)$$



# Pneumatic Joints



# Hardware Platforms



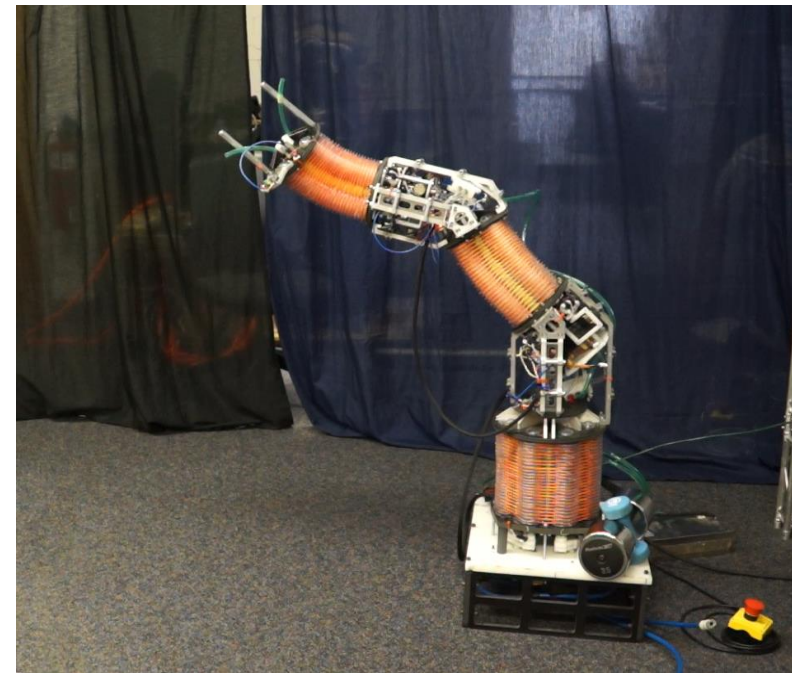
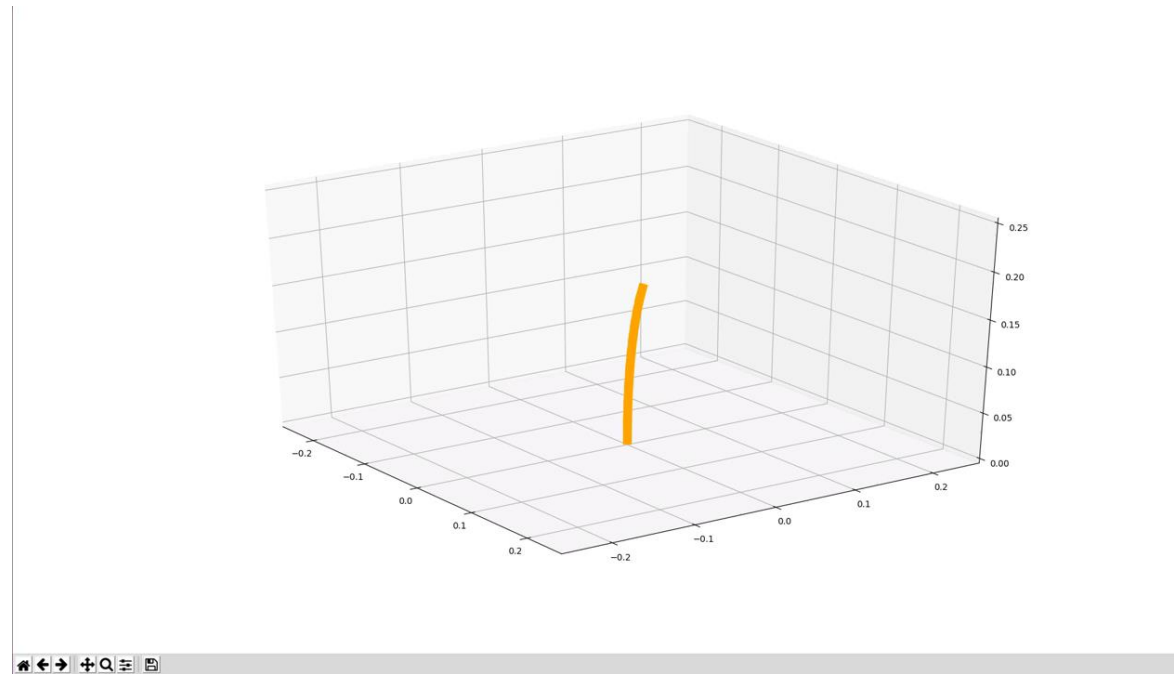
# Three different modeling approaches to enable control

1. First principles modeling (based on physical phenomenon).
2. Adapting unknown terms in dynamics.
3. Fully learned models.

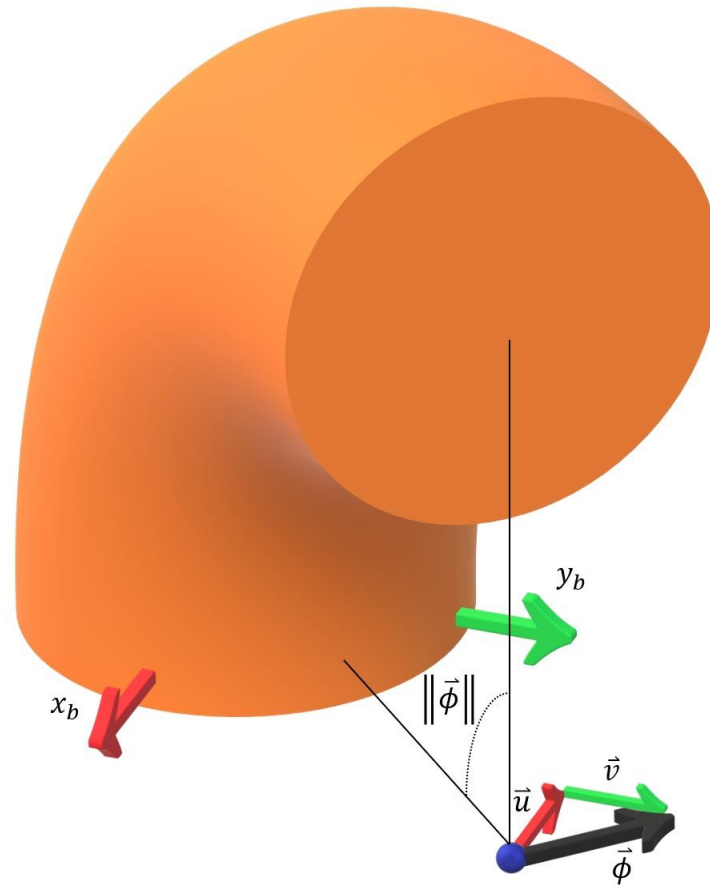


# First Principles Modeling for Control

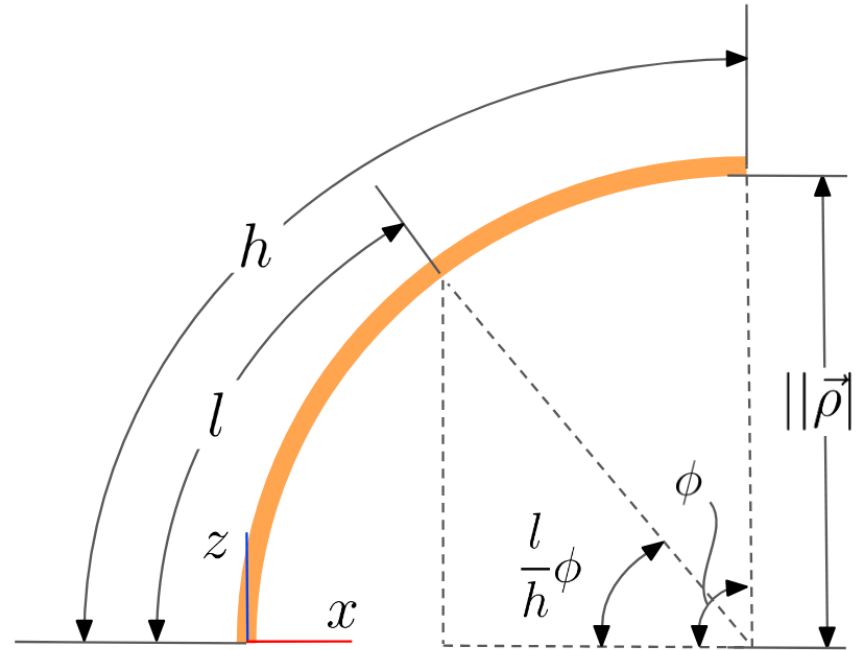
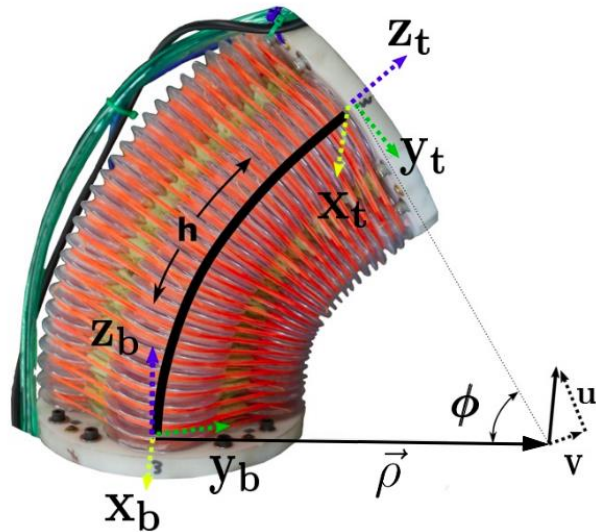
# First Principles Models



# Kinematic Modeling



# Kinematic Modeling



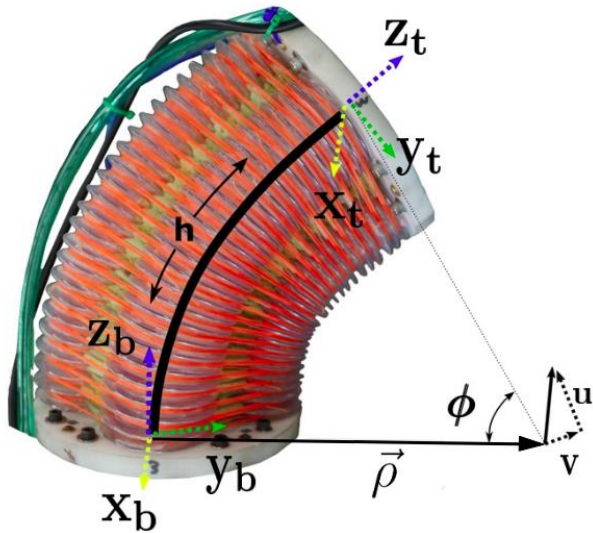
$$\begin{bmatrix} v_l \\ u_l \\ 0 \end{bmatrix} = \frac{l}{h} \begin{bmatrix} v_h \\ u_h \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \dot{v}_l \\ \dot{u}_l \\ 0 \end{bmatrix} = \frac{l}{h} \begin{bmatrix} \dot{v}_h \\ \dot{u}_h \\ 0 \end{bmatrix}$$

Assumptions:

- Piecewise constant curvature

# Pneumatically Actuated Robot Dynamics (using Euler-Lagrange Dynamics)



$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = Q$$



$$M(q)\ddot{q} + C(\dot{q}, q)\dot{q} + g(q) = \tau$$

$$L = T - V$$

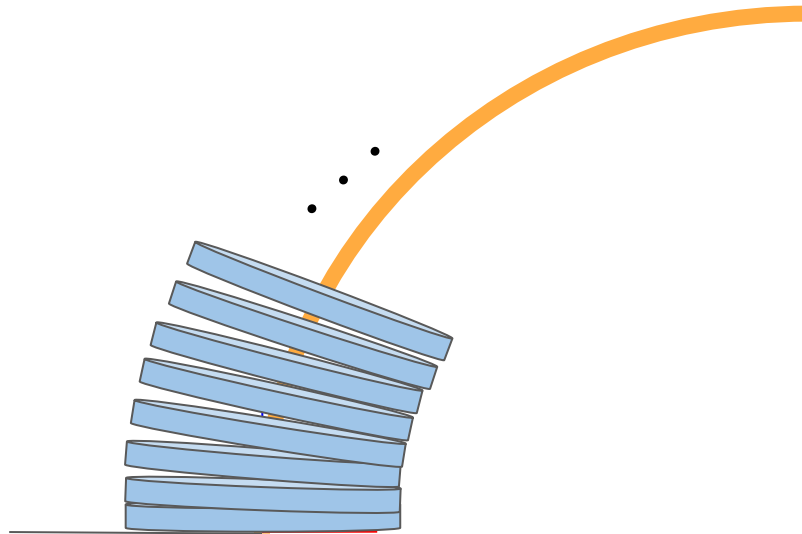
Where:

- L is called the Lagrangian
- T is kinetic energy
- V is potential energy

$$q = \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} u_h \\ v_h \end{bmatrix}$$

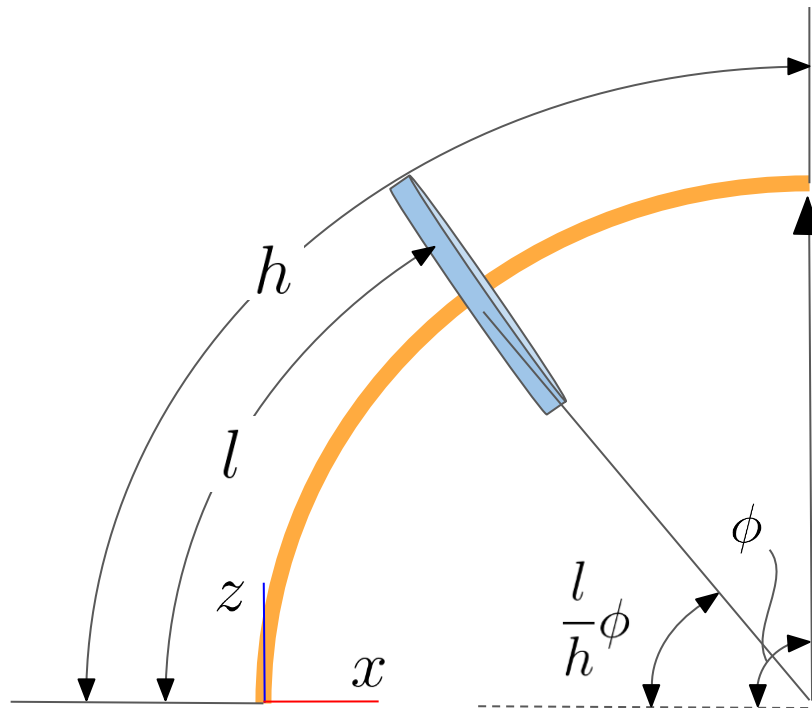
$$\dot{q} = \begin{bmatrix} \dot{u} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} \dot{u}_h \\ \dot{v}_h \end{bmatrix}$$

# Model Mass as a Series of Disks



Let each disk represent a section of the continuum joint with some mass and rotational inertia.

# Kinematics (location and velocity) of each Disk



$$\begin{bmatrix} v_l \\ u_l \\ 0 \end{bmatrix} = \frac{l}{h} \begin{bmatrix} v_h \\ u_h \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \dot{v}_l \\ \dot{u}_l \\ 0 \end{bmatrix} = \frac{l}{h} \begin{bmatrix} \dot{v}_h \\ \dot{u}_h \\ 0 \end{bmatrix}$$

# Soft Robot Modeling and Estimation

$$J_{weighted} = \begin{bmatrix} \sqrt{\mu} J \dot{p}_{l,x} \\ \sqrt{\mu} J \dot{p}_{l,y} \\ \sqrt{\mu} J \dot{p}_{l,z} \\ \frac{\sqrt{\mu} r}{2} J \omega_{l,x} \\ \frac{\sqrt{\mu} r}{2} J \omega_{l,y} \\ \frac{\sqrt{\mu} r}{\sqrt{2}} J \omega_{l,z} \end{bmatrix}$$

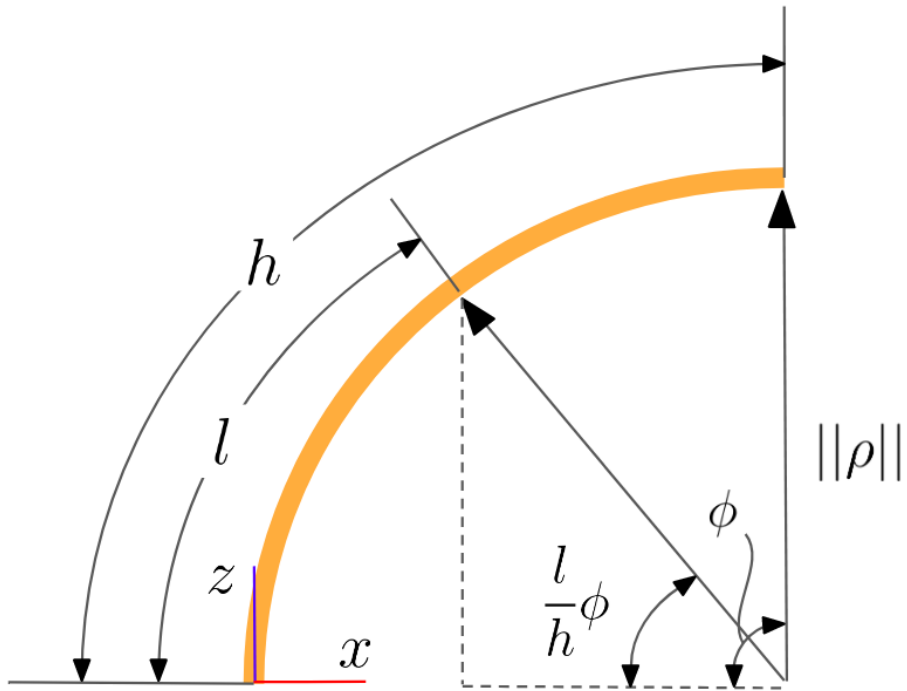
Now T (kinetic energy) for a single disk can be written as a function of time derivative of the generalized coordinates:

$$T_l = \frac{1}{2} \dot{q}^T J_{weighted}(u_l, v_l, l)^T J_{weighted}(u_l, v_l, l) \dot{q} dl$$

$$\begin{bmatrix} \dot{v}_l \\ \dot{u}_l \\ 0 \end{bmatrix} = \frac{l}{h} \begin{bmatrix} \dot{v}_h \\ \dot{u}_h \\ 0 \end{bmatrix}$$



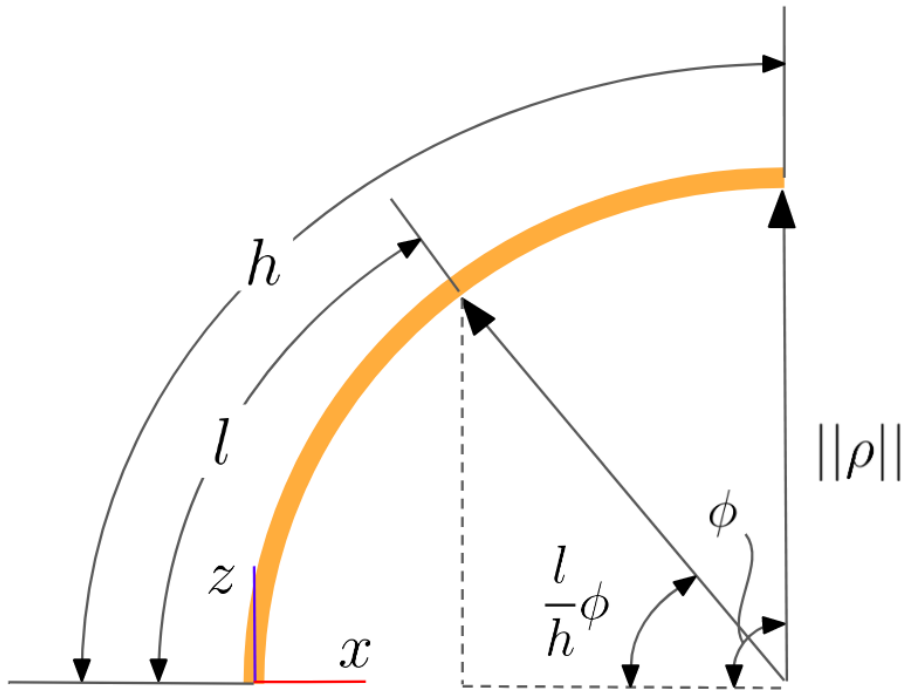
# Soft Robot Modeling and Estimation



$$T = \frac{1}{2} \dot{q}^T \left[ \int_0^h J_{weighted}(u_l, v_l, l)^T J_{weighted}(u_l, v_l, l) dl \right] \dot{q}$$

Dissertation - Hyatt, P. E. (2020). Robust Real-Time Model Predictive Control for High Degree of Freedom Soft Robots.  
(with a journal paper under review)

# Soft Robot Modeling and Estimation



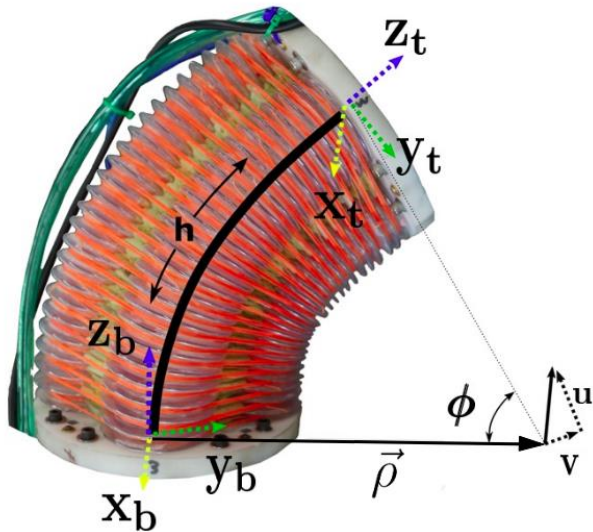
For potential energy  $P$ , we just need the center of mass of each continuum segment as a function of our generalized coordinates:

$$\vec{p} = \frac{h}{\phi^2} \begin{bmatrix} (\phi - \sin(\phi)) \frac{v}{\phi} \\ (\phi - \sin(\phi)) \frac{-u}{\phi} \\ (1 - \cos(\phi)) \end{bmatrix}$$

If “ $G$ ” is the gravity vector expressed in the same frame as “ $p$ ”, then potential energy “ $V$ ” can be written as:

$$V = \vec{p} \cdot \vec{G}$$

# Pneumatically Actuated Robot Dynamics (using Euler-Lagrange Dynamics)



$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = Q$$



$$M(q)\ddot{q} + C(\dot{q}, q)\dot{q} + g(q) = \tau$$

$$M(q)\ddot{q} + C(\dot{q}, q)\dot{q} + g(q) = K_{pressure} p + K_{spring} q + K_{damper} \dot{q}$$

$$q = \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} u_h \\ v_h \end{bmatrix}$$

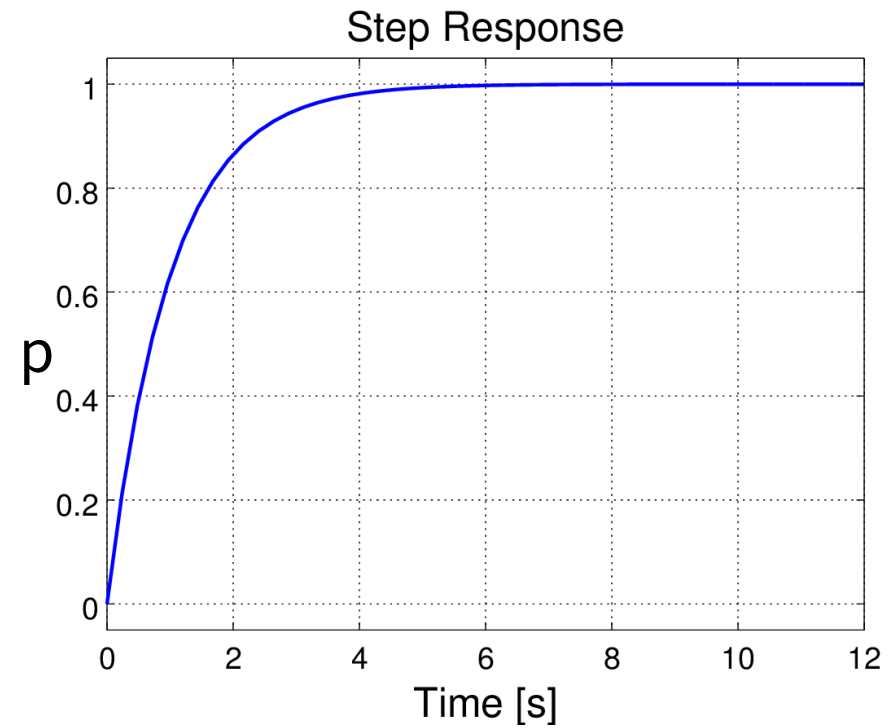
$$\dot{q} = \begin{bmatrix} \dot{u} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} \dot{u}_h \\ \dot{v}_h \end{bmatrix}$$

Where:

- M is the mass matrix in the generalized coordinate space
- C is the Coriolis and centripetal terms
- g is the torque due to gravity
- Tau is the friction AND parasitic torque AND actuation torque from pressure

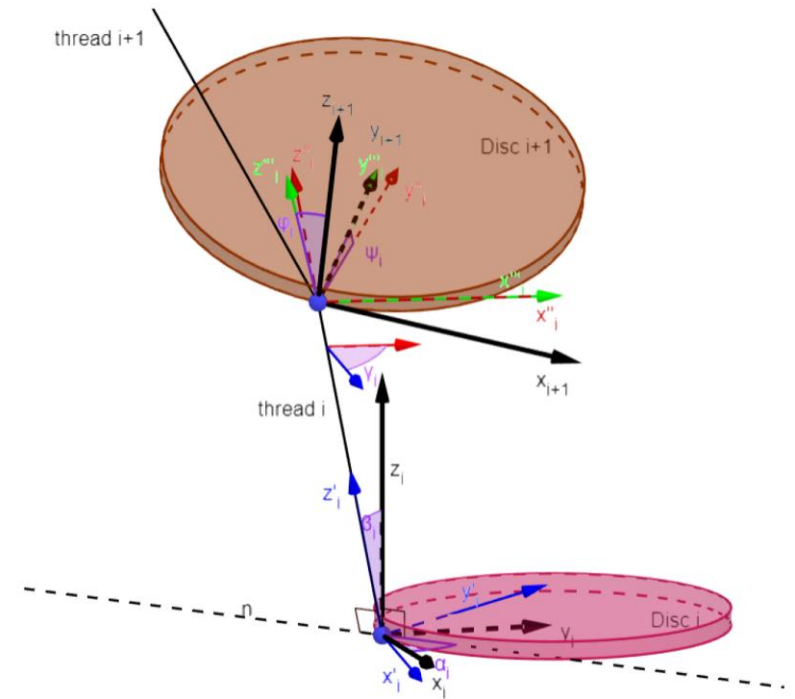
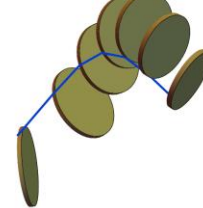
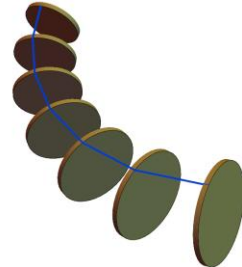
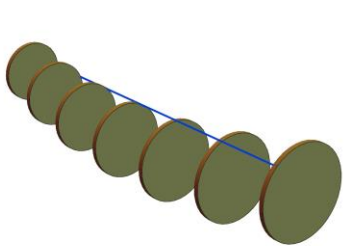
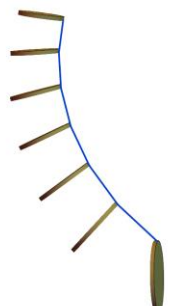
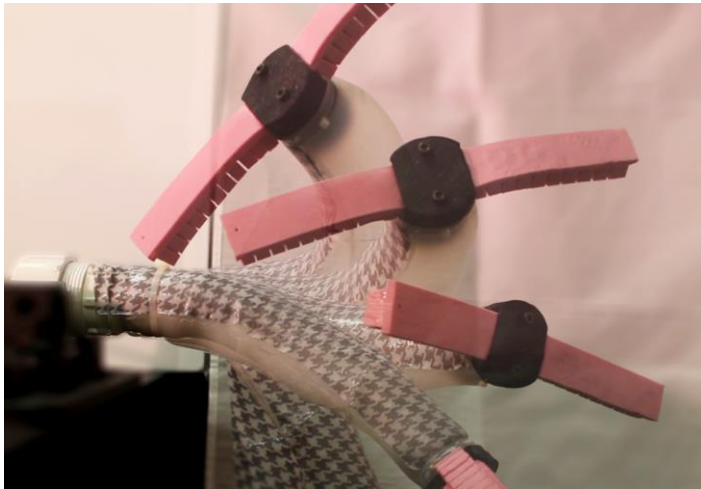
# Pressure Dynamics

$$\dot{p} = \alpha(p_{ref} - p)$$



# Thread-Disk Model Preview

- In collaboration with Dr. Joshua Schultz at the University of Tulsa
- Although lumped models, extra degrees of freedom allow it to bend, buckle, and twist.

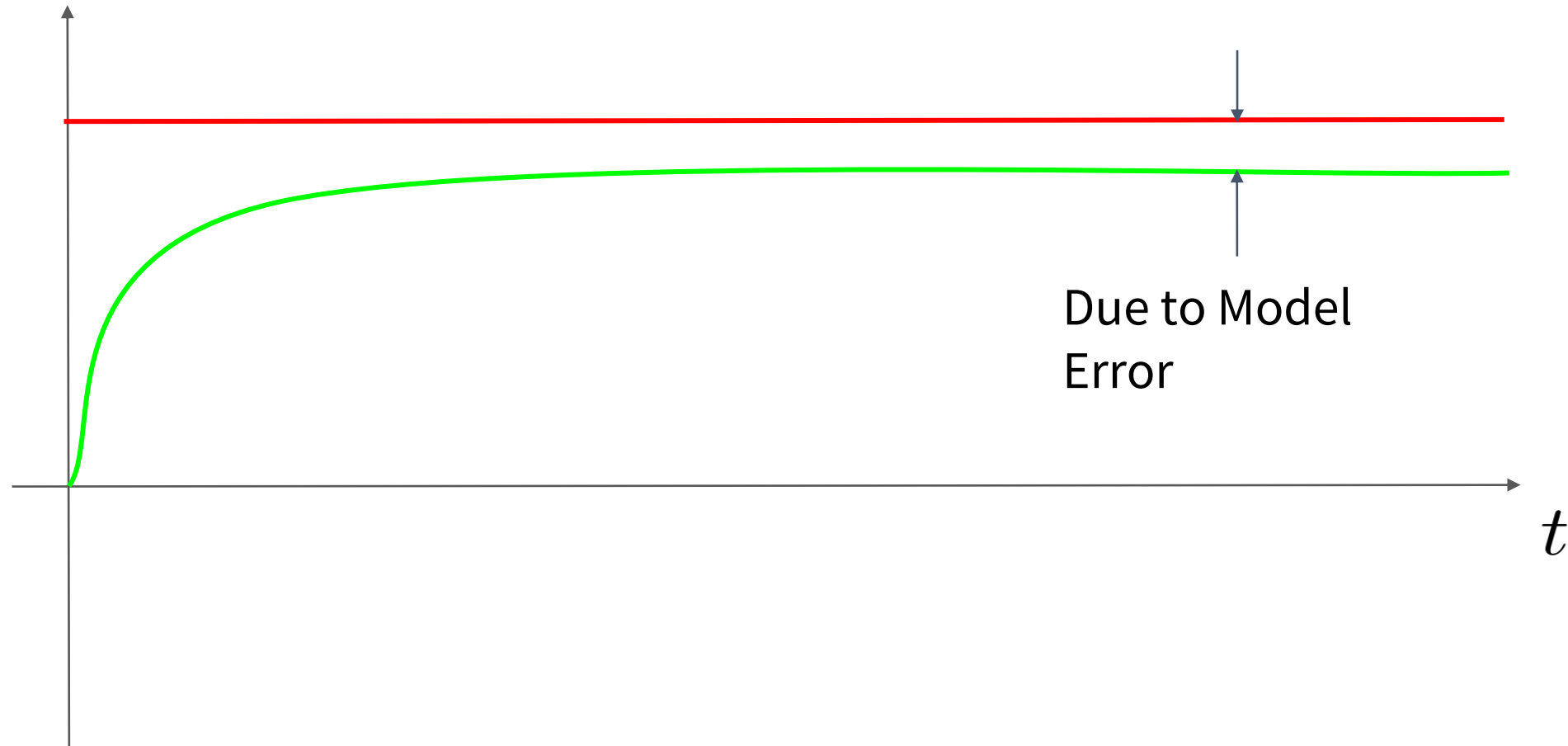


# First Principles Modeling for Control

- Pros –
  - Intuition behind model and its meaning for control
  - Can easily add terms (like friction or parasitic torque)
  - Can look at dominant terms in model
  - Clear methods for system ID
  - Likely makes proving stability much easier
- Cons -
  - No guarantee that your model is accurate enough for control
  - Can take lots of work to develop a completely new model (disk-thread)
    - And may still not represent the system well
  - Still requires collecting data to do system ID (although not as much as ML)
  - May not be tractable for control

# Adapting Unknown Dynamics

# Model-based Control Still Gives Steady State Error





# Model Reference Adaptive Control (MRAC)

Let control be the following:

$$\tau = M\ddot{q}_{ref} + C\dot{q} + g$$

$$M\ddot{q} + C\dot{q} + g = \tau$$

$$\ddot{q} = M^{-1}[\tau - C\dot{q} - g]$$

$$\ddot{q} = M^{-1}[(M\ddot{q}_{ref} + C\dot{q} + g) - C\dot{q} - g]$$

$$\ddot{q} = \cancel{M^{-1}(M\ddot{q}_{ref})} + \cancel{M^{-1}(C\dot{q} - C\dot{q})} + \cancel{M^{-1}(g - g)}$$

$$\ddot{q} = \ddot{q}_{ref}$$

# Model Reference Adaptive Control (MRAC)

$$\tau = M\ddot{q}_{ref} + C\dot{q} + g$$

$$\tau = Y(q, \dot{q}, \dot{q}_{ref}, \ddot{q}_{ref})a$$

- We can rewrite our dynamics model in terms of a regressor matrix, and an unknown set of parameters in the vector “a”
- Only the regressor (Y) is needed
  - Usually starts with form from first principles model, but can add whatever terms we think are relevant.
- Provably stable convergence to the reference trajectory
- Given enough control authority, you can make your system behave like whatever “reference” system you chose

# MRAC – what's the catch?

What about terms that don't show up in the regressor?  
(because we don't know how to express them?)

$$M\ddot{q} + C\dot{q} + g + ??? = \tau$$



$$\ddot{q} = \ddot{q}_{ref} + M^{-1}???$$

# MRAC + MPC = MRPAC

Can we combine the robustness of model-based MPC, with the adaptation of unknown terms from MRAC?

$$\dot{x} = Ax + Bu + w$$

Disturbances to linear model in general when doing MPC alone.

$$\dot{x} = Ax + Bu + (w + Ya)$$

Disturbances that include a dynamics term based on our regressor AND the adaptive control term "a" for MRAC+MPC

# Comparison of Control with Three Different Methods

- Model predictive control (MPC, model-based approximation for optimal control)
- Model reference adaptive control (MRAC)
- Model reference adaptive control + MPC (MRPAC)

## **Comparisons:**

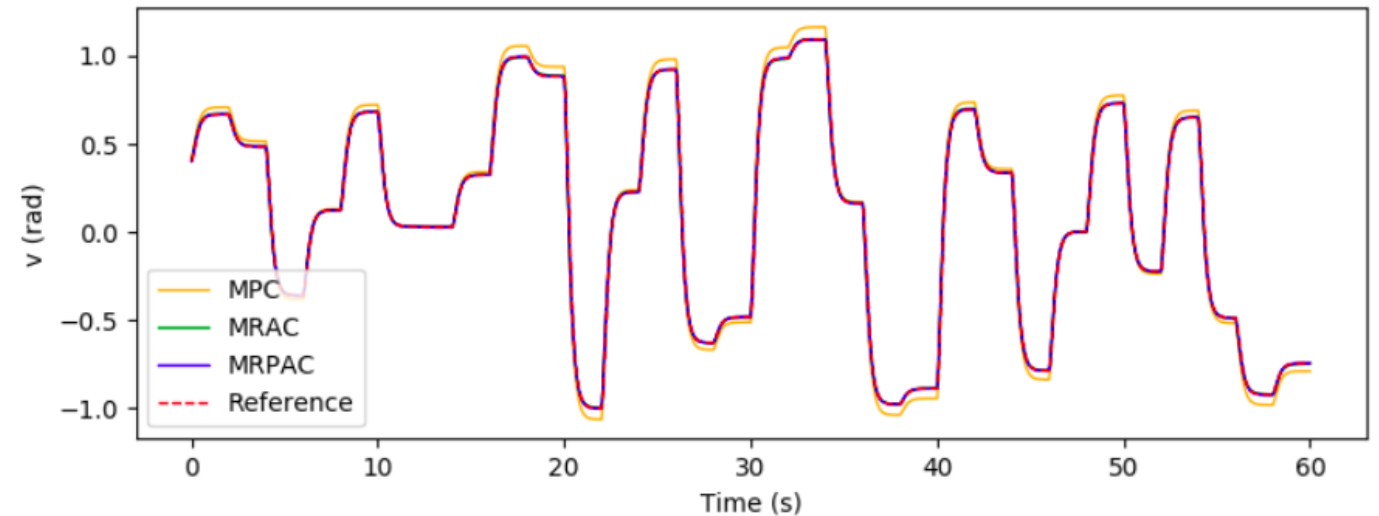
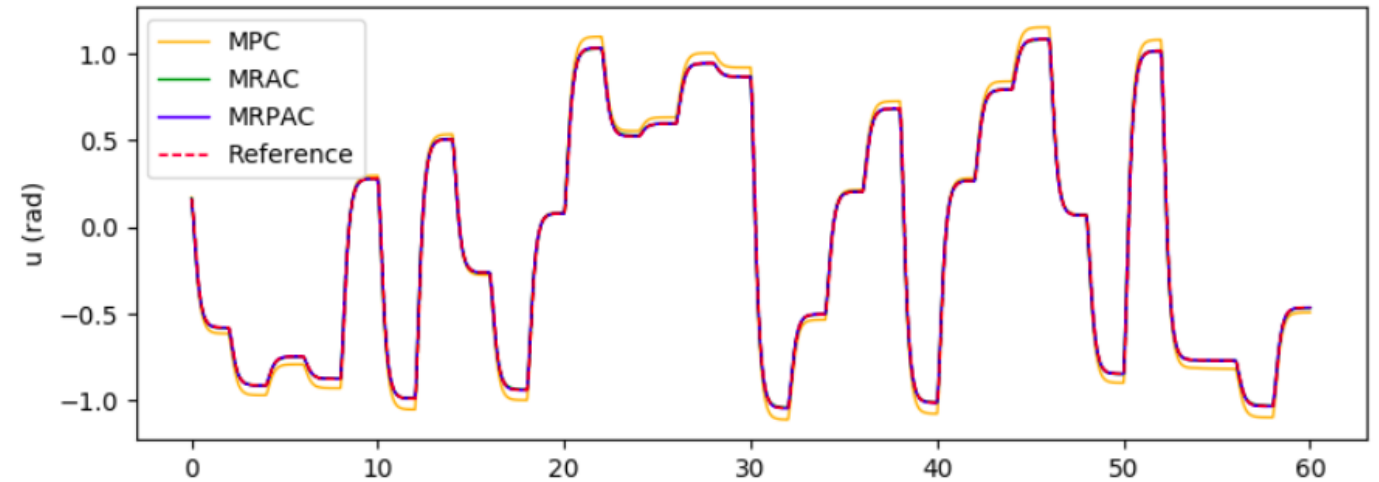
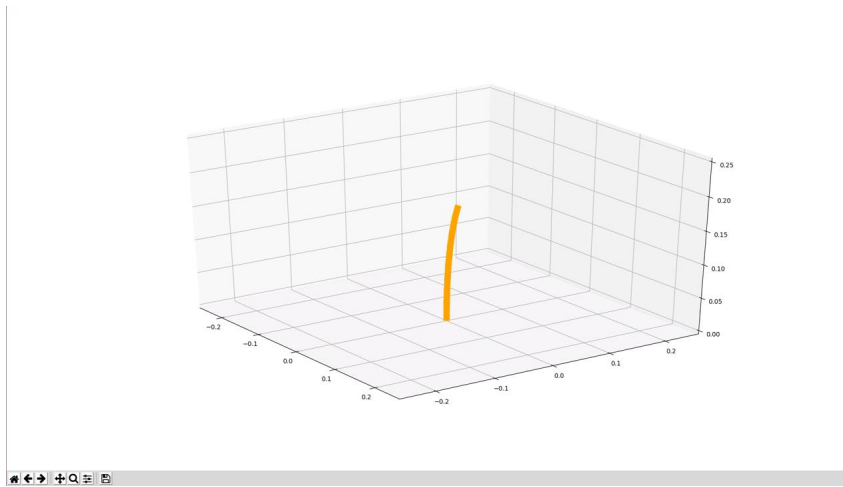
1. with a parameter mismatch (in inertia) – sim
2. with a model mismatch (in spring return equilibrium position) – sim
3. on real hardware where there is both parameter and model mismatch

# Effect of Model Mismatch

(significant error in inertia)

## Model parameter error

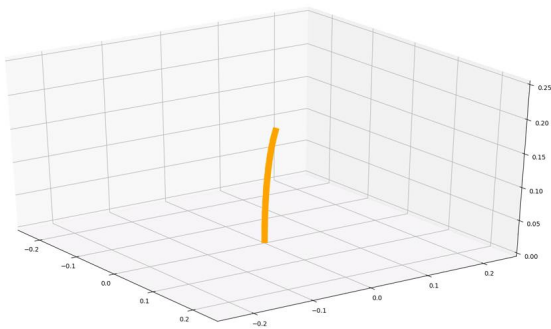
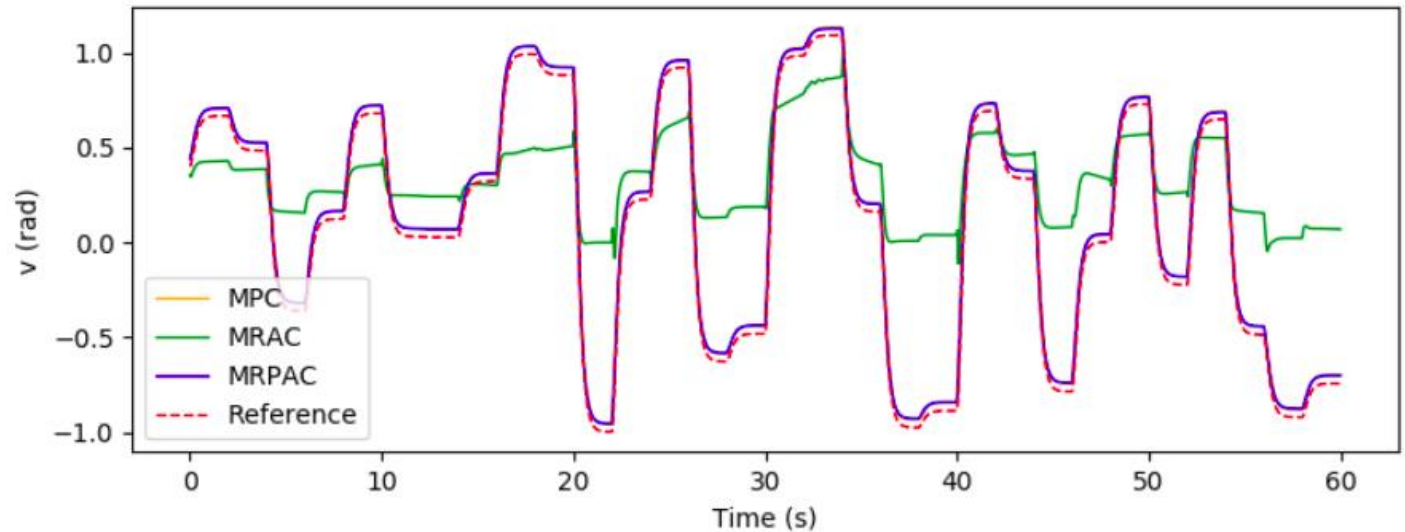
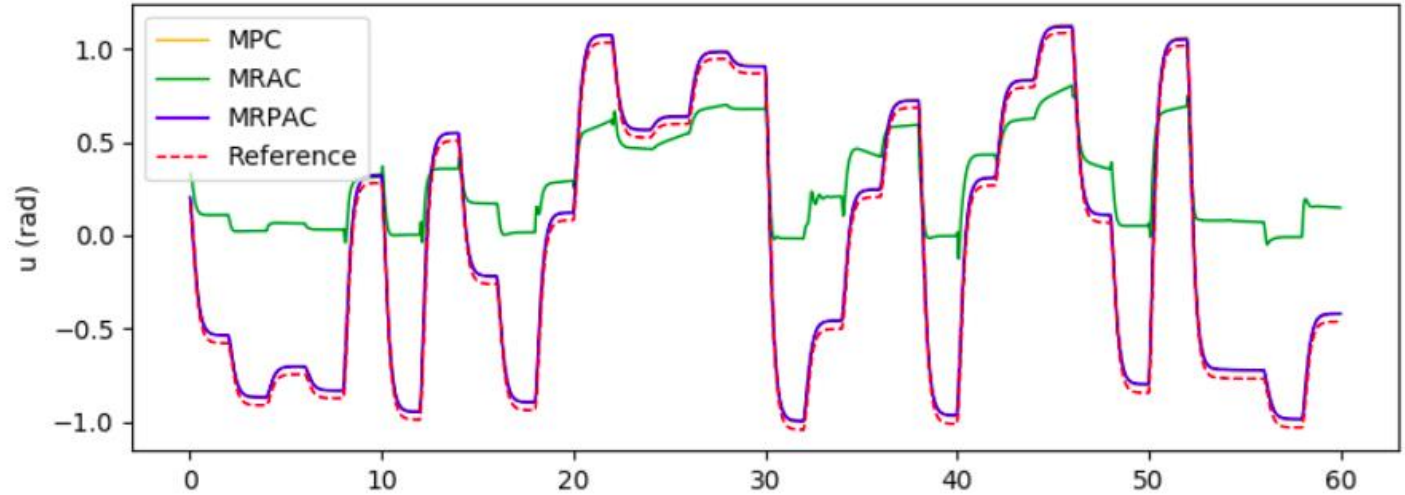
(e.g incorrect estimates of length, mass, stiffness, and/or inertia)



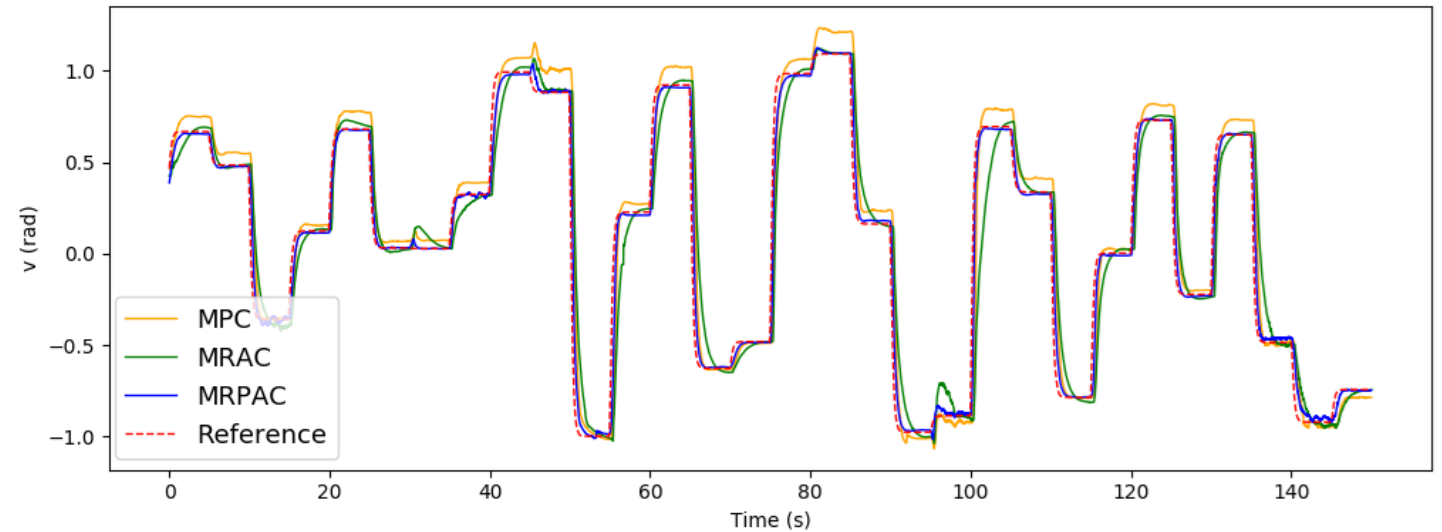
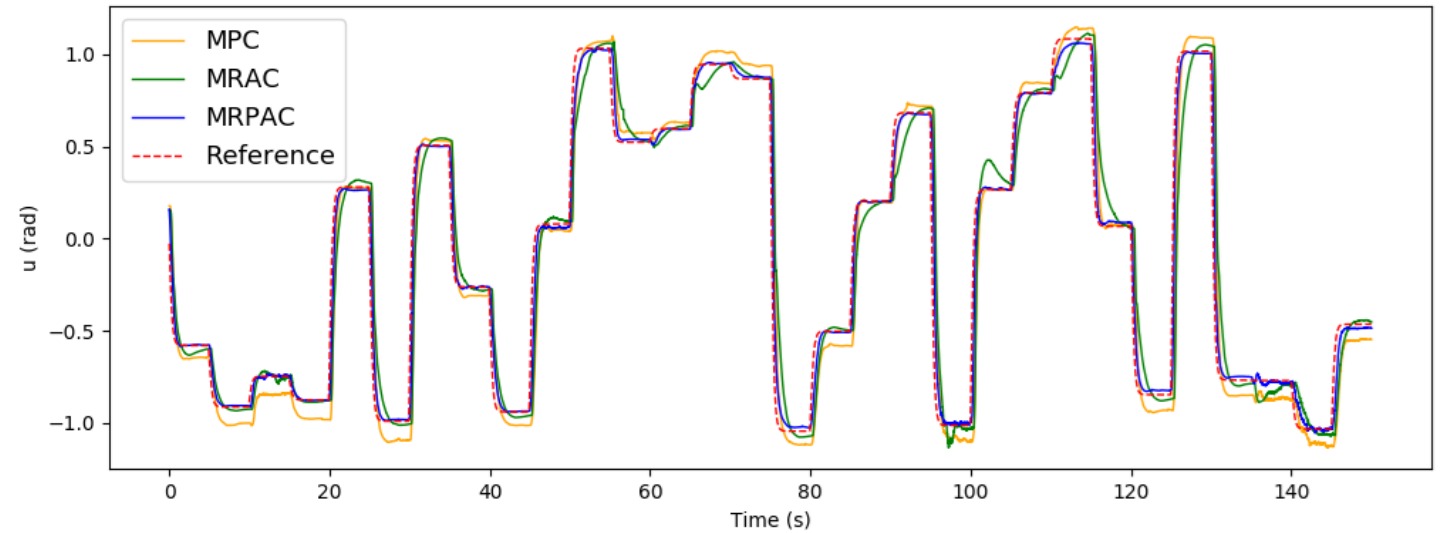
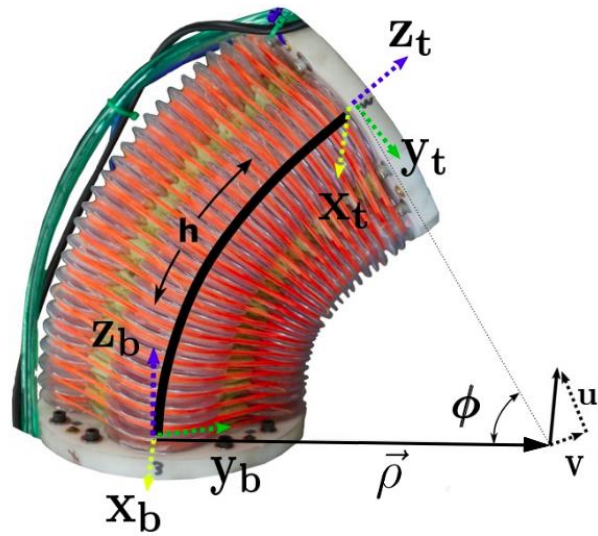
# Effect of Structural Mismatch (error in equilibrium config)

## Offset Force/Torque

(e.g. wrong spring equilibrium value)



# Hardware Results



Dissertation - Hyatt, P. E. (2020). Robust Real-Time Model Predictive Control for High Degree of Freedom Soft Robots. (with a journal paper under review)

Terry, J. S., Whitaker, J., Beard, R. W., & Killpack, M. D. (2019, October). Adaptive Control of Large-Scale Soft Robot Manipulators With Unknown Payloads. In *ASME 2019 Dynamic Systems and Control Conference*. American Society of Mechanical Engineers Digital Collection.

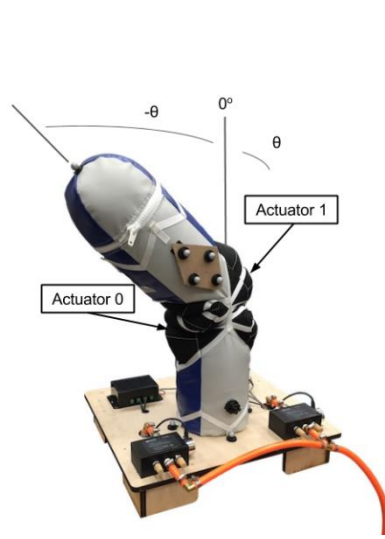


# Online Adaptive Modeling for Control

- Many of the same pros and cons of modeling based on first principles ...
- Pros -
  - Don't need to accurately know all terms in model (we'll adapt them over time)
  - Can adapt parameters based on tracking error (MRAC) instead of model error alone
- Cons -
  - Still have to identify or guess the form of the regressor terms
  - Adaptive control alone is not robust to unmodeled terms (like hysteresis, friction, choked/unchoked flow in pneumatics)

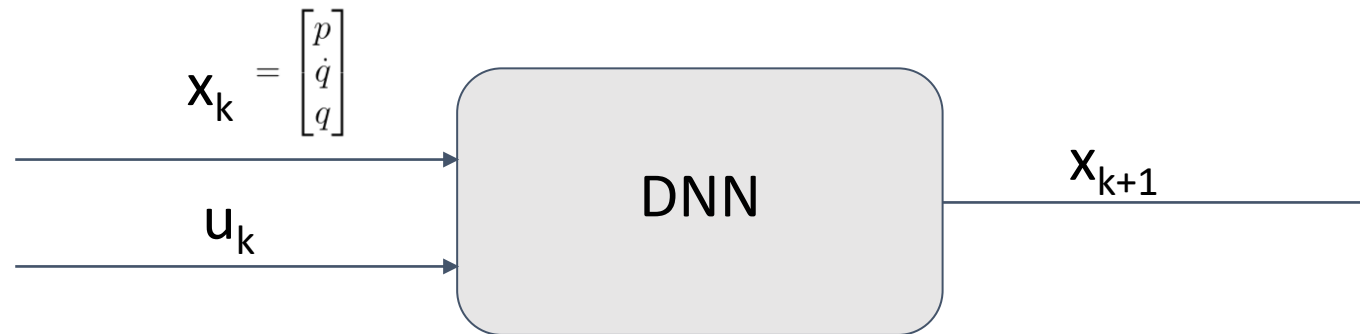
# Fully Learned Models

# Current State-of-the-Art for Large Scale Soft Robot Manipulation



- Results involve years of work using first principle models, hand-tuned controllers
- THAT'S NO GOOD if ...
  - Every soft robot platform is different from another
  - Every platform still requires significant system identification

# Learned Discrete-Time Models



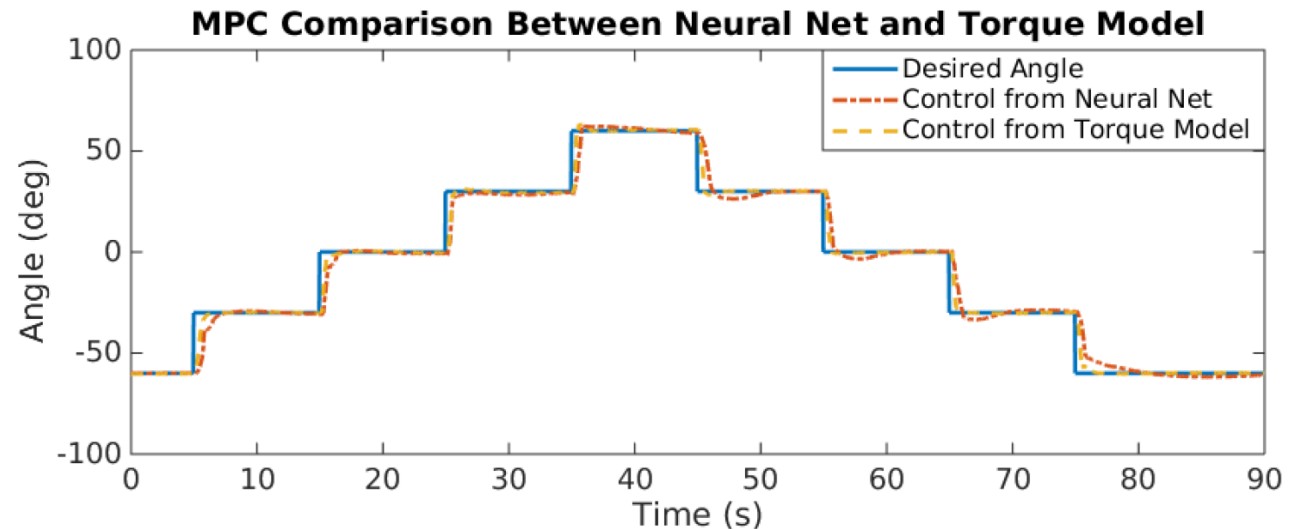
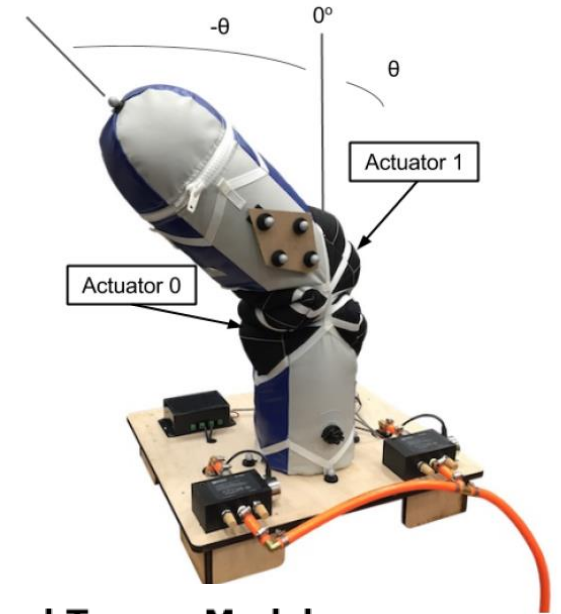
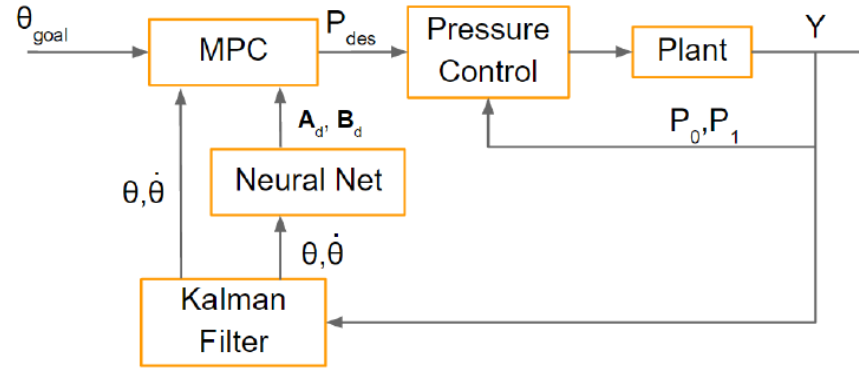
# Learned DNN models for control

$$\mathbf{x}[k+1] = f(\mathbf{x}[k], \mathbf{u}[k])$$

$$\mathbf{A}_d = \begin{bmatrix} \frac{\partial f_1}{\partial \dot{\theta}} & \frac{\partial f_1}{\partial \theta} & \frac{\partial f_1}{\partial P_0} & \frac{\partial f_1}{\partial P_1} \\ \frac{\partial f_2}{\partial \dot{\theta}} & \frac{\partial f_2}{\partial \theta} & \frac{\partial f_2}{\partial P_0} & \frac{\partial f_2}{\partial P_1} \\ \frac{\partial f_3}{\partial \dot{\theta}} & \frac{\partial f_3}{\partial \theta} & \frac{\partial f_3}{\partial P_0} & \frac{\partial f_3}{\partial P_1} \\ \frac{\partial f_4}{\partial \dot{\theta}} & \frac{\partial f_4}{\partial \theta} & \frac{\partial f_4}{\partial P_0} & \frac{\partial f_4}{\partial P_1} \\ \frac{\partial \theta}{\partial \dot{\theta}} & \frac{\partial \theta}{\partial \theta} & \frac{\partial \theta}{\partial P_0} & \frac{\partial \theta}{\partial P_1} \end{bmatrix}$$

$$\mathbf{B}_d = \begin{bmatrix} \frac{\partial f_1}{\partial P_{0,des}} & \frac{\partial f_1}{\partial P_{1,des}} \\ \frac{\partial f_2}{\partial P_{0,des}} & \frac{\partial f_2}{\partial P_{1,des}} \\ \frac{\partial f_3}{\partial P_{0,des}} & \frac{\partial f_3}{\partial P_{1,des}} \\ \frac{\partial f_4}{\partial P_{0,des}} & \frac{\partial f_4}{\partial P_{1,des}} \\ \frac{\partial P_0}{\partial P_{0,des}} & \frac{\partial P_0}{\partial P_{1,des}} \\ \frac{\partial P_1}{\partial P_{0,des}} & \frac{\partial P_1}{\partial P_{1,des}} \end{bmatrix}$$

$$\mathbf{x}[k+1] = \mathbf{A}_d \mathbf{x}[k] + \mathbf{B}_d \mathbf{u}[k]$$



# Extension of Learned DNN models for Multi-DoF Control

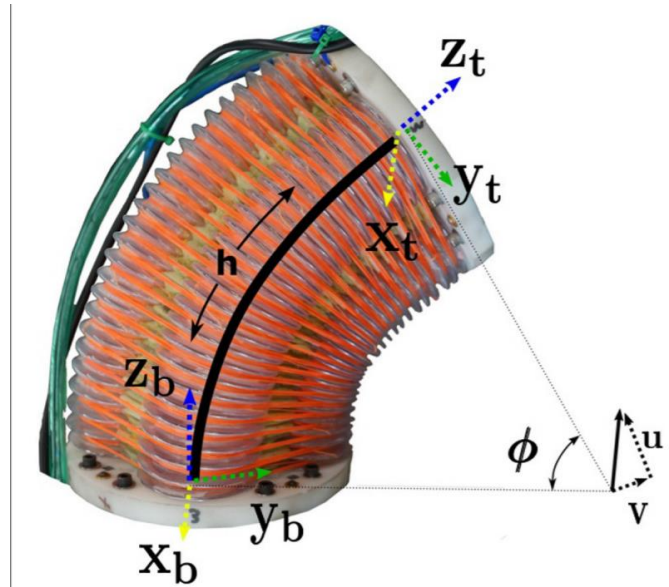
- Use DNN to approximate –  $\dot{q}_{k+1} = f(\mathbf{x}_k, \mathbf{u}_k)$ .
- Then dynamic model can be written as follows:

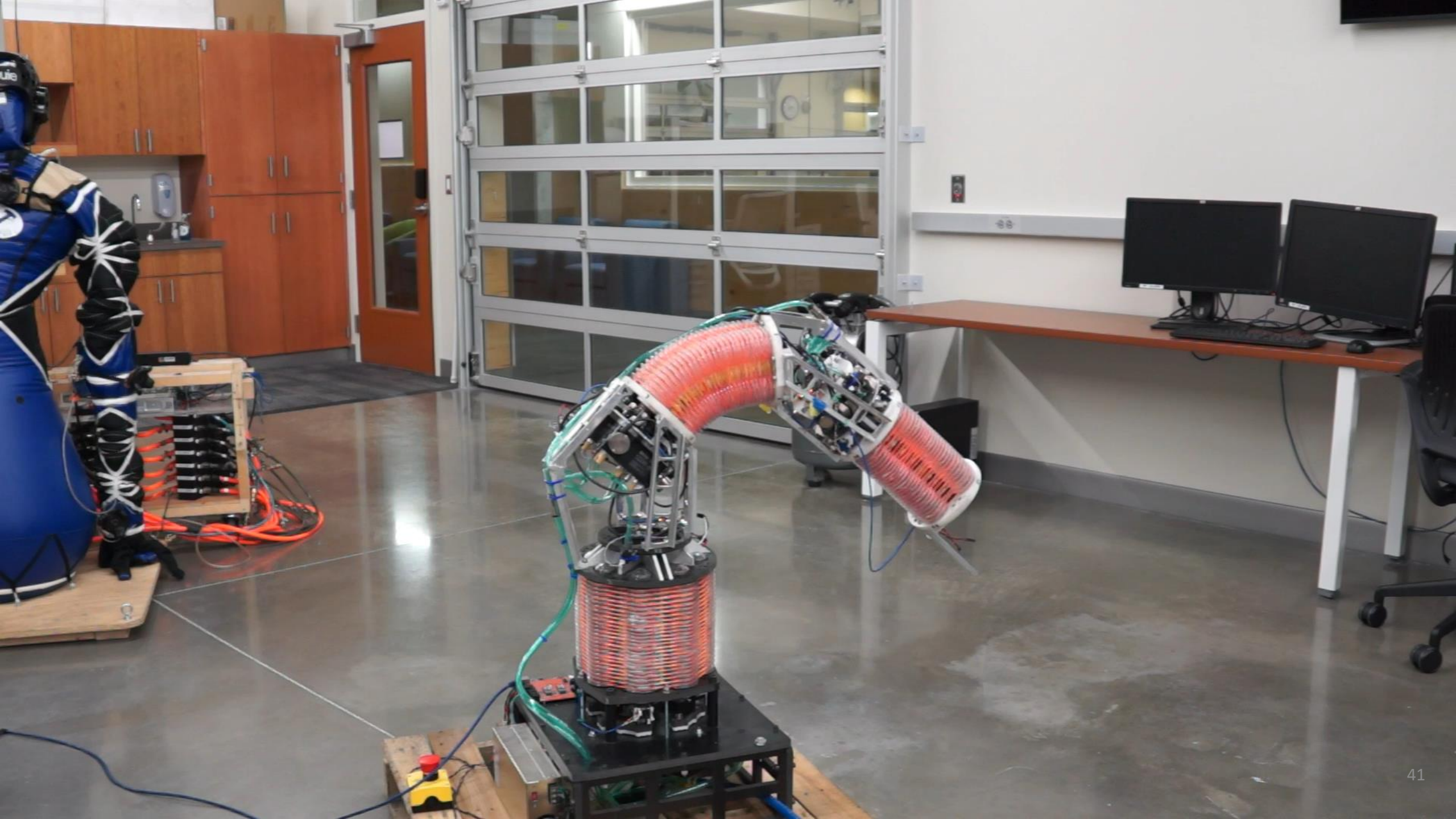
$$\mathbf{x}_{k+1} = \mathbf{A}_d(\mathbf{x}_k - \mathbf{x}_k) + \mathbf{B}_d(\mathbf{u}_k - \mathbf{u}_k) + \mathbf{w}_d$$

$$\mathbf{A}_d = \begin{bmatrix} (I - \alpha \Delta t) & 0 & 0 \\ \frac{\partial f}{\partial p_k} & \frac{\partial f}{\partial \dot{q}_k} & \frac{\partial f}{\partial q_k} \\ \frac{\partial f}{\partial p_k} \frac{\Delta t}{2} & (\frac{\partial f}{\partial \dot{q}_k} + I) \frac{\Delta t}{2} & \frac{\partial f}{\partial q_k} \frac{\Delta t}{2} + I \end{bmatrix}$$

$$\mathbf{B}_d = \begin{bmatrix} \alpha \Delta t \\ \frac{\partial f}{\partial p_{ref,k}} \\ 0 \end{bmatrix}$$

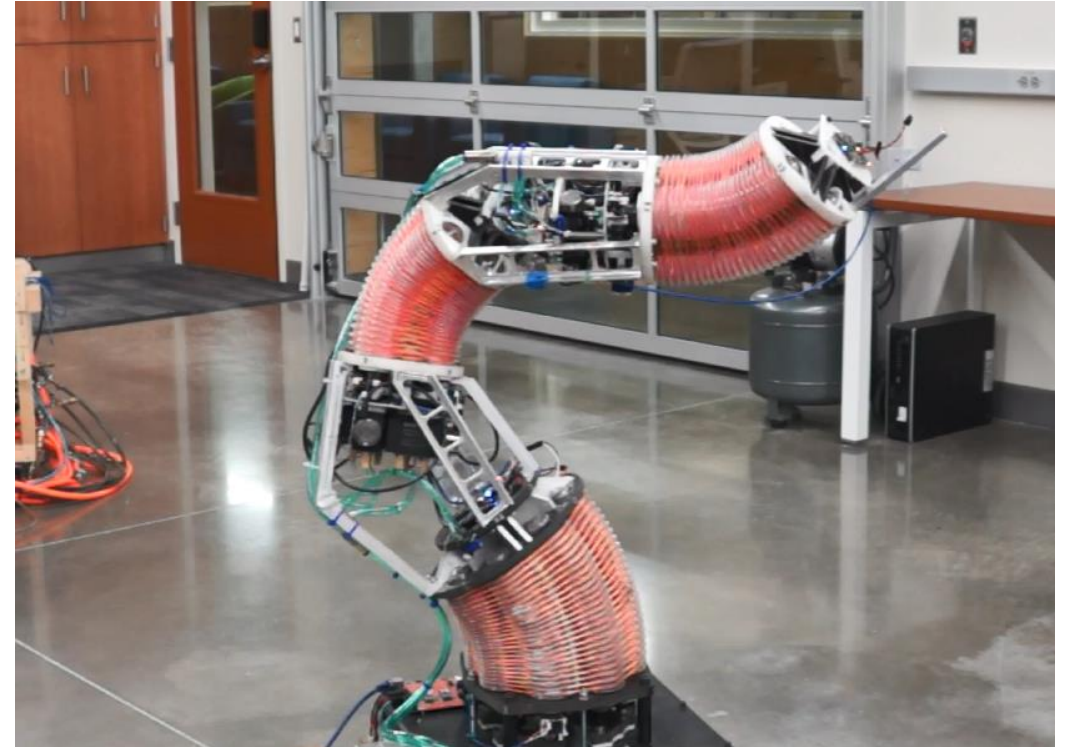
$$\mathbf{w}_d = \begin{bmatrix} p_0 \\ f(\mathbf{x}_0, \mathbf{u}_0) \\ \frac{\Delta t}{2} \dot{q}_0 + q_0 \end{bmatrix}$$





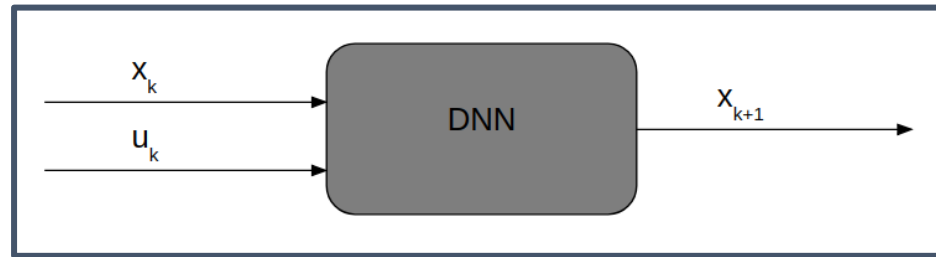
# Limitations

1. Using convex solver, still slow if want more joints
2. Not really making use of fast GPU evaluation
3. Cannot represent nonlinear effects over a long time horizon

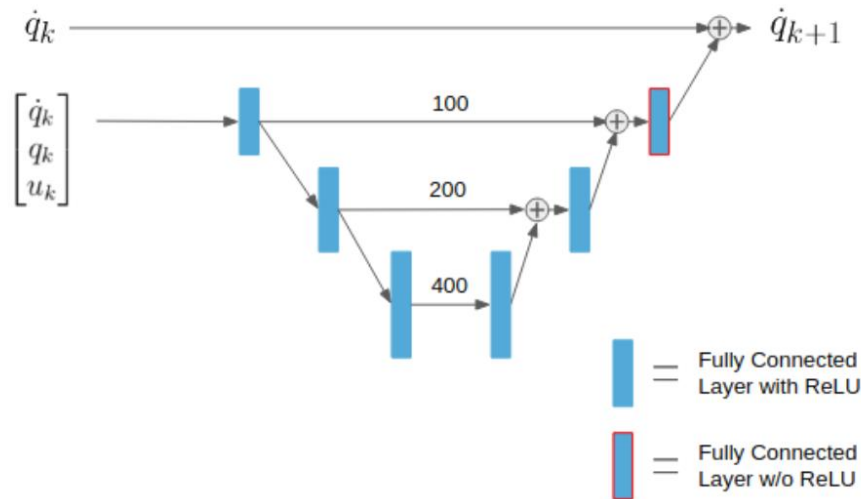
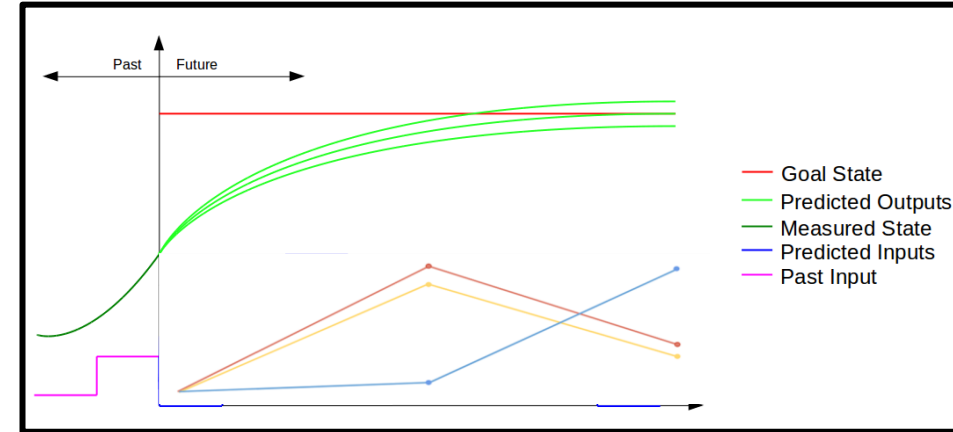




# Linear/Nonlinear EMPC



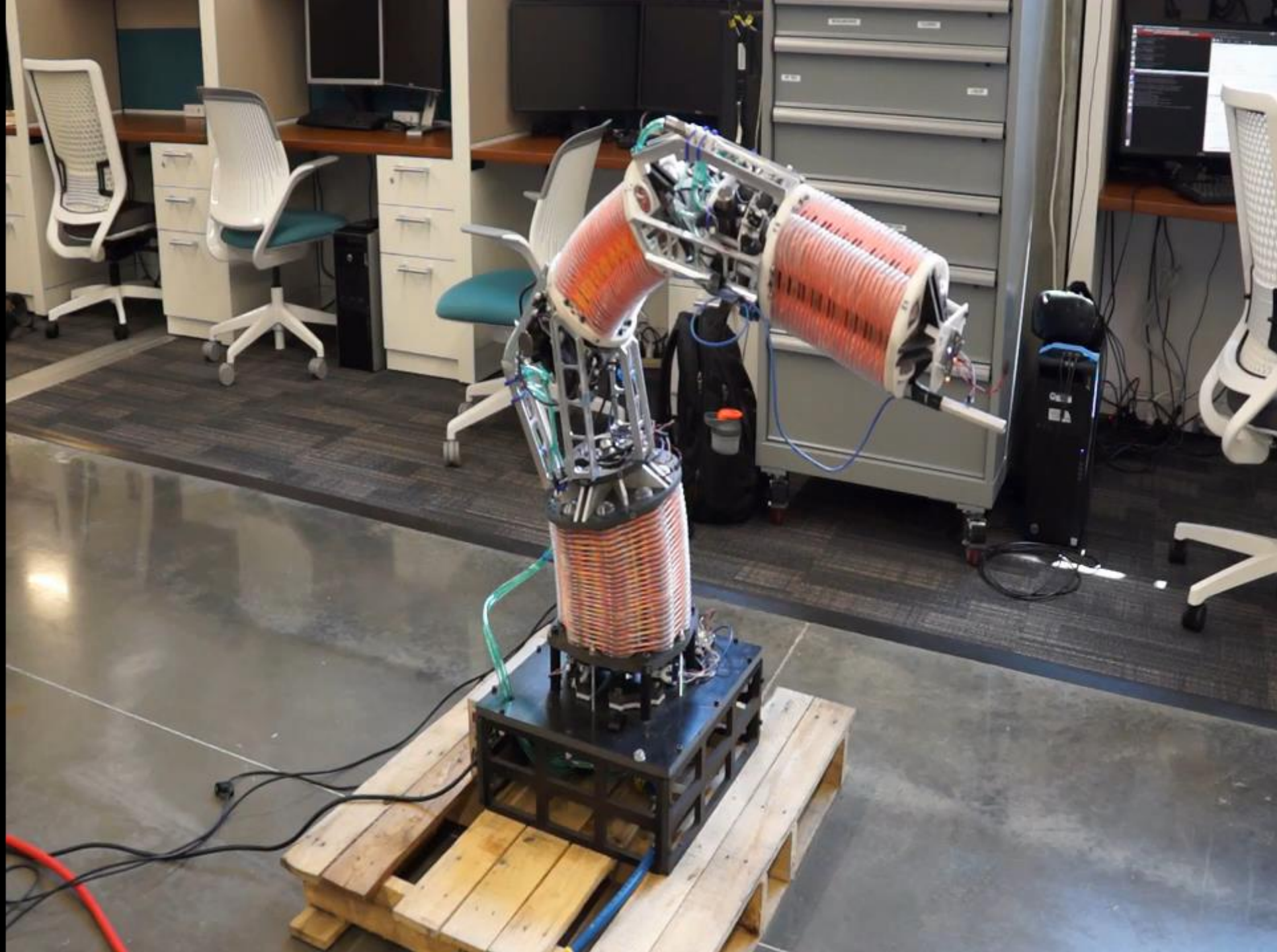
+



Hyatt, P., & Killpack, M. D. (2019). Real-Time Nonlinear Model Predictive Control Using a Graphics Processing Unit. *IEEE Robotics and Automation Letters* 2020.

Hyatt, P., & Killpack, M. D. (2017, November). Real-time evolutionary model predictive control using a graphics processing unit. In *2017 IEEE-RAS 17th International Conference on Humanoid Robotics (Humanoids)* (pp. 569-576). IEEE.

Hyatt, P., Williams, C. S., & Killpack, M. D. (2020). Parameterized and GPU-Parallelized Real-Time Model Predictive Control for High Degree of Freedom Robots. *arXiv preprint arXiv:2001.04931*.



# Learned Models for Control

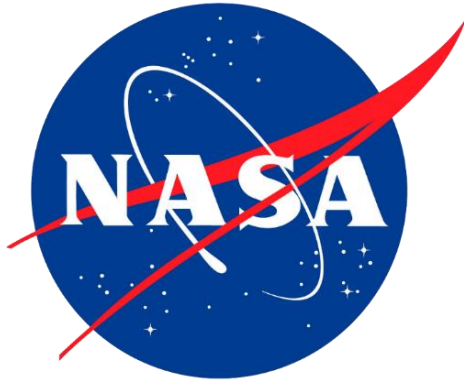
- Pros

- Just collect data and hit "go"
- Can use GPUs for fast evaluation (and solution for MPC)
- Can easily represent nonlinear models

- Cons

- Questions about scalability for realtime evaluation vs accuracy
- Picking the right DNN architecture matters a lot
- Collecting "useful" data to get good models for control is difficult
- Lack of intuition (explainable AI could help)
- No guarantees about stability or even generality of model

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Otherlab

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Thank You