Simplifying Kinematic Models through Geometric Constraints

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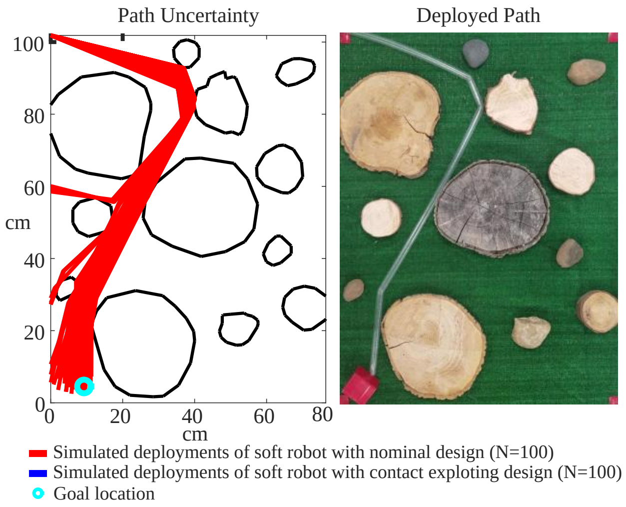
Design through geometric constraints and simplified kinematic models

- High link between **morphology** and **behavior** of soft robots
- Modeling can be a useful design tool, if models are simple enough
- Geometric constraints create simple building blocks that can be repeated to create complex behaviors
- Combination of simplified modeling and geometric constraints yields design principles for creating complex and useful kinematics from compliant systems

Applying geometric constraints to simplify kinematic models

Obstacle interaction to decrease uncertainty



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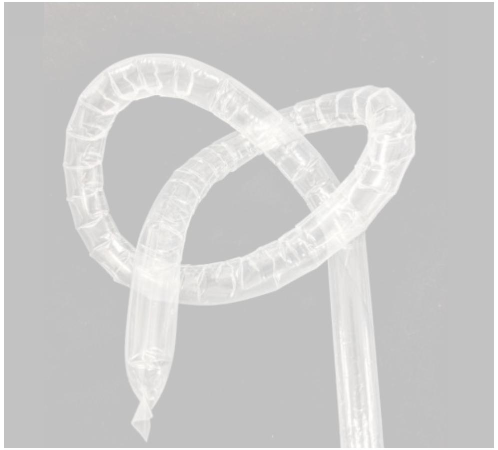
Geometric models of general actuation



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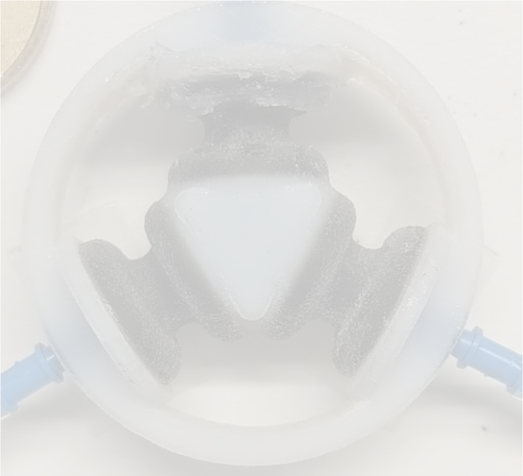
Design of soft delta mechanisms



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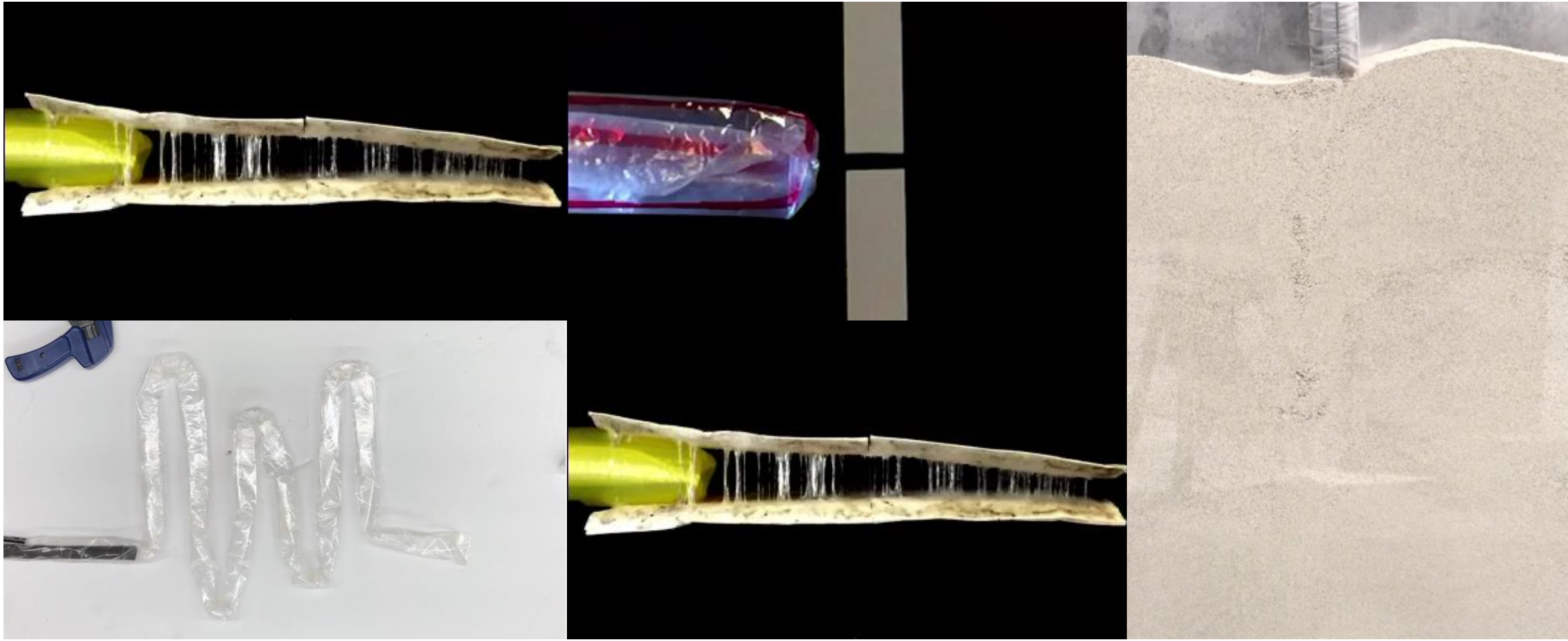
Introduction

Pneumatic Tip-Extending Soft Robot

Growing Affects Environmental Interactions

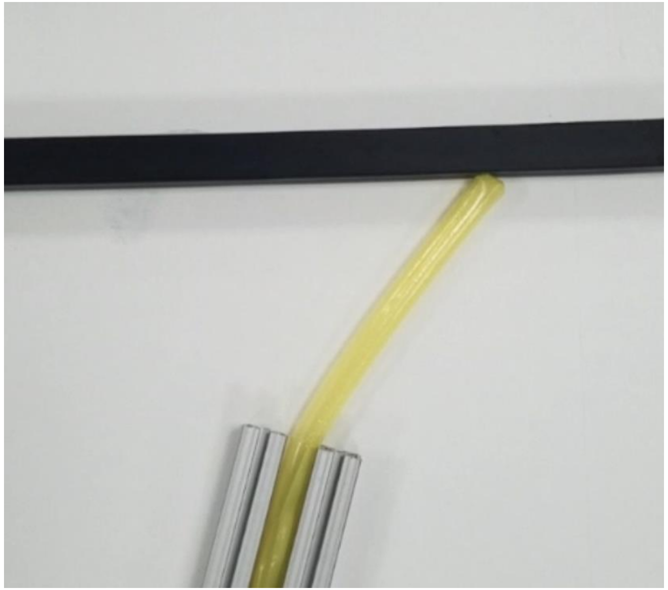


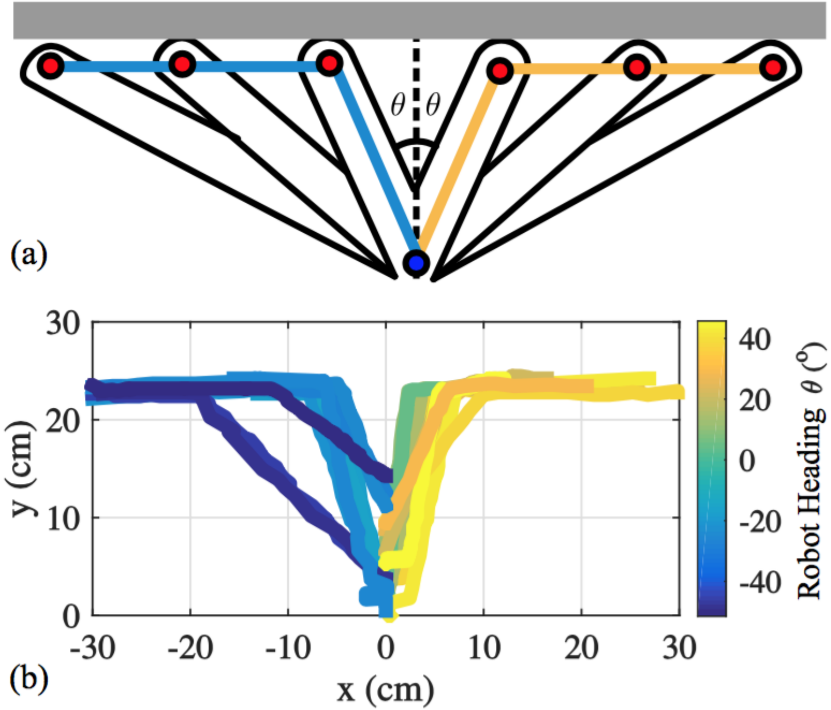
Question: How do we harness environment interactions to improve navigation?

Useful Obstacle Interaction Behavior

Environment passively guides the robot

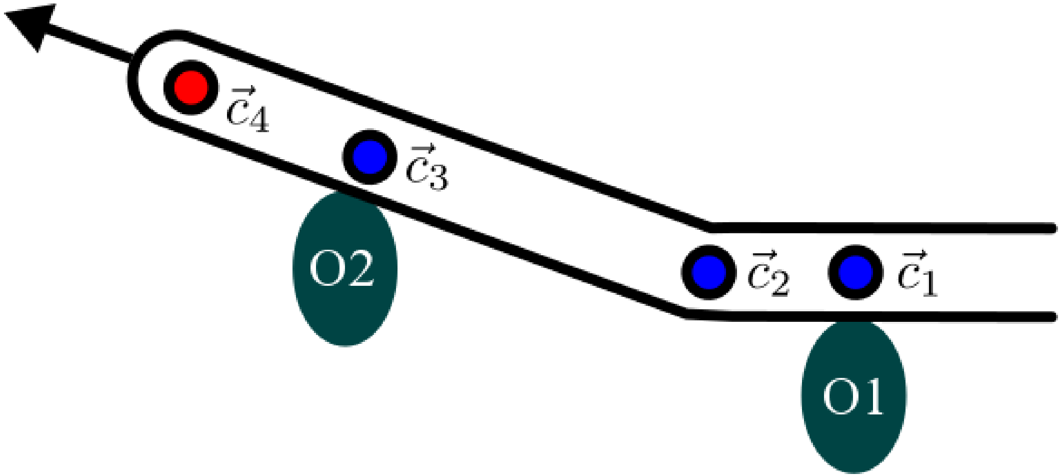
Varying the Initial Contact Angle





Building an Obstacle-Aided Navigation Model: Robot State

- Robot state = pivot points
- Pivot points (two types):
  - Obstacle contact
  - Pre-made turn
- Obstacle contacts added as encountered



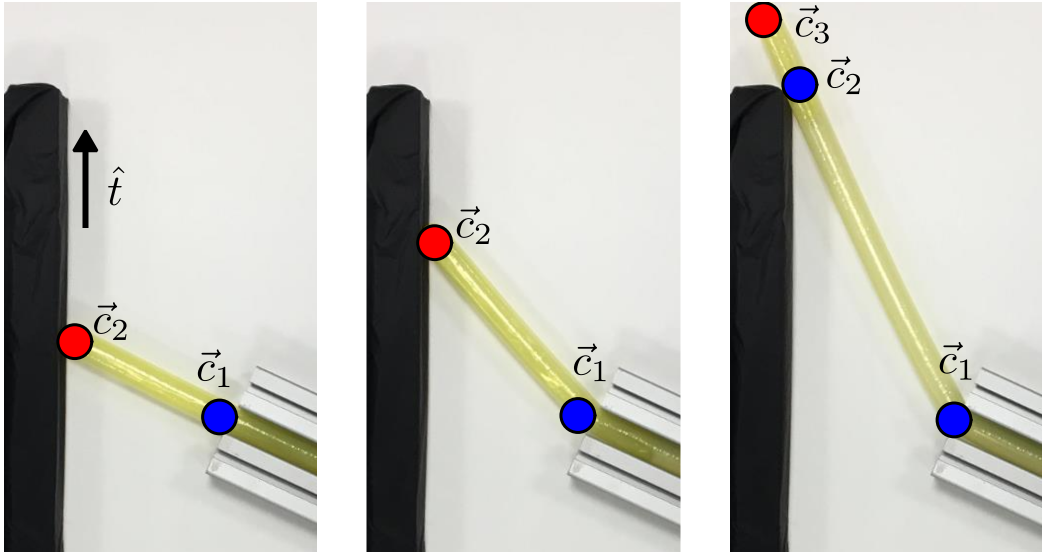
Building an Obstacle-Aided Navigation Model: Kinematics

Free-Growth Differential Kinematics:

$$\dot{\vec{c}}_n = u\frac{1}{||\vec{c}_n - \vec{c}_{n-1}||}(\vec{c}_n - \vec{c}_{n-1})$$

Obstacle Contact Differential Kinematics:

$$\dot{\vec{c}}_n = u\frac{||\vec{c}_n - \vec{c}_{n-1}||}{\hat{t}\cdot(\vec{c}_n - \vec{c}_{n-1})}\hat{t}$$



$u\left(\frac{m}{s}\right)$ is controlled growth rate

Model Validation: Navigation by Obstacles Only

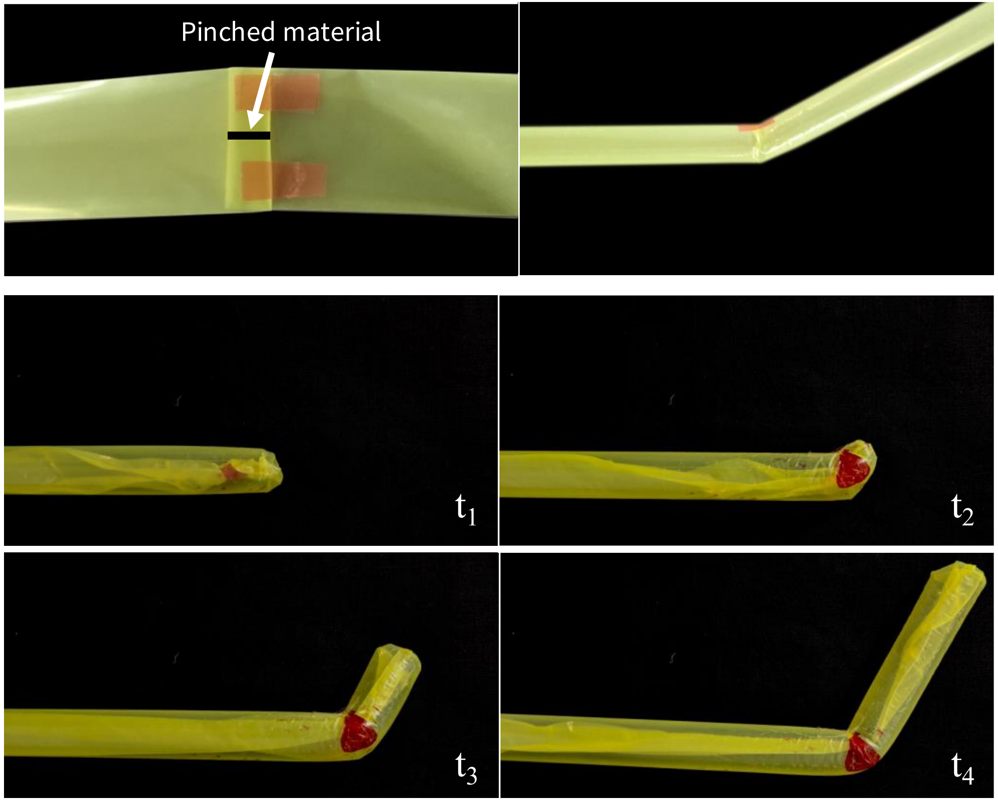
Obstacle-Aided Navigation of a Soft Growing Robot

Joseph D. Greer[1], Laura H. Blumenschein[1], Allison M. Okamura[1], and Elliot W. Hawkes[2]

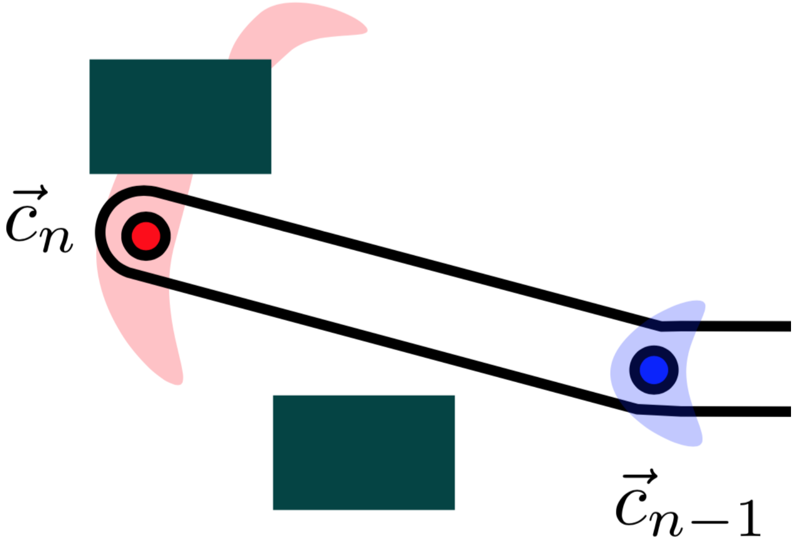
[1] Stanford University
[2] University of California, Santa Barbara

Adding in Steering

Uniformly shorten one side



Planning Robot Paths to Intelligently Use Obstacle Contacts



Nominal design: $(\underline{l}_1, \underline{\theta}_1, \ldots, \underline{l}_m, \underline{\theta}_m)$

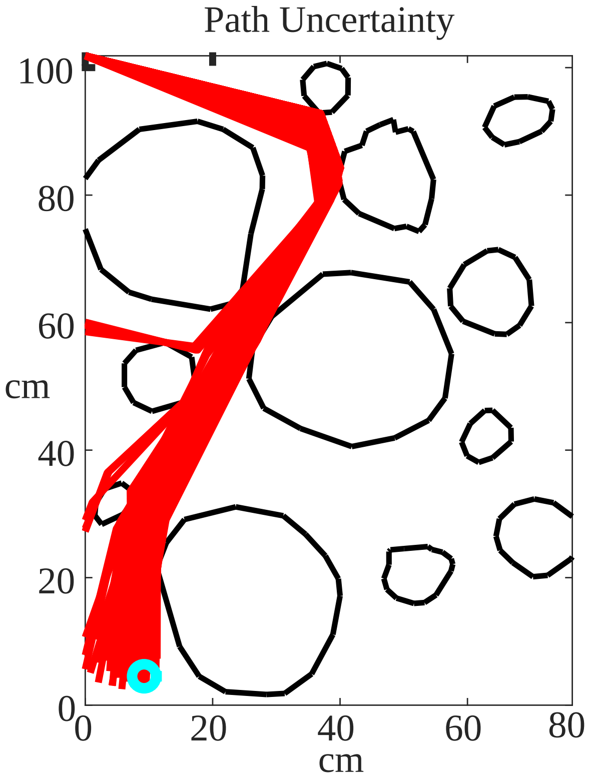
Manufacturing Error ↓

Built design: $(\underset{\sim}{l}_1, \underset{\sim}{\theta}_1, \ldots, \underset{\sim}{l}_m, \underset{\sim}{\theta}_m)$

Planning Objective:
Find nominal design with highest expectation of reaching desired target given obstacle interactions

Planning In a Cluttered Environment

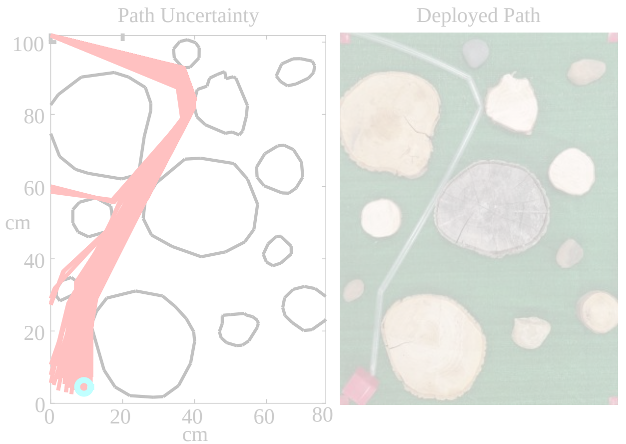
Planning In a Cluttered Environment



Simulated deployments of soft robot with nominal design (N=100)
Simulated deployments of soft robot with contact exploting design (N=100)
Goal location

Applying geometric constraints to simplify kinematic models

Obstacle interaction to decrease uncertainty



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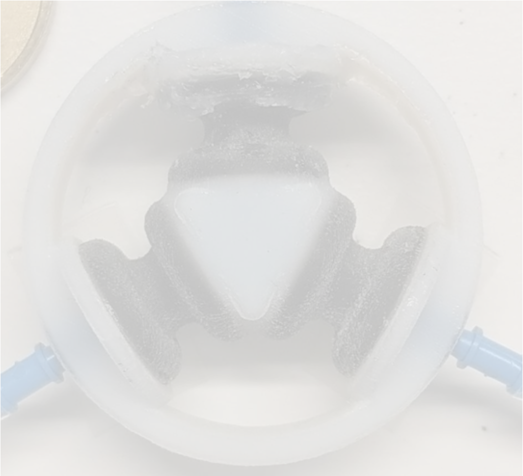

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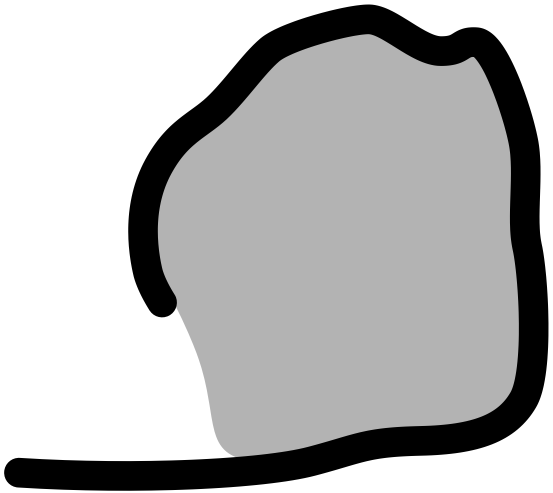
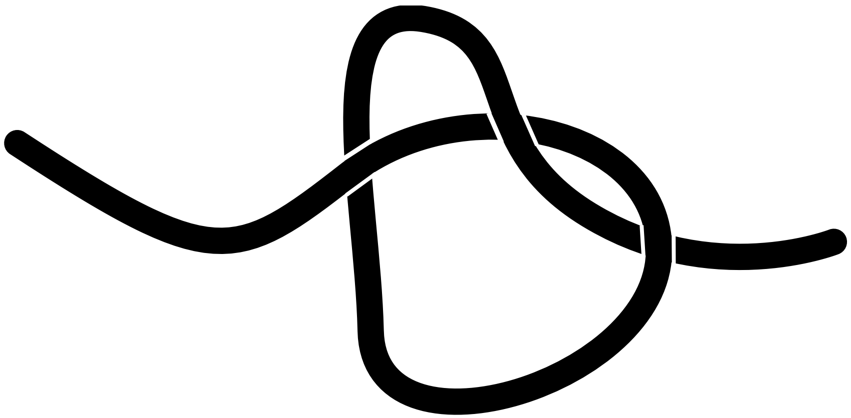
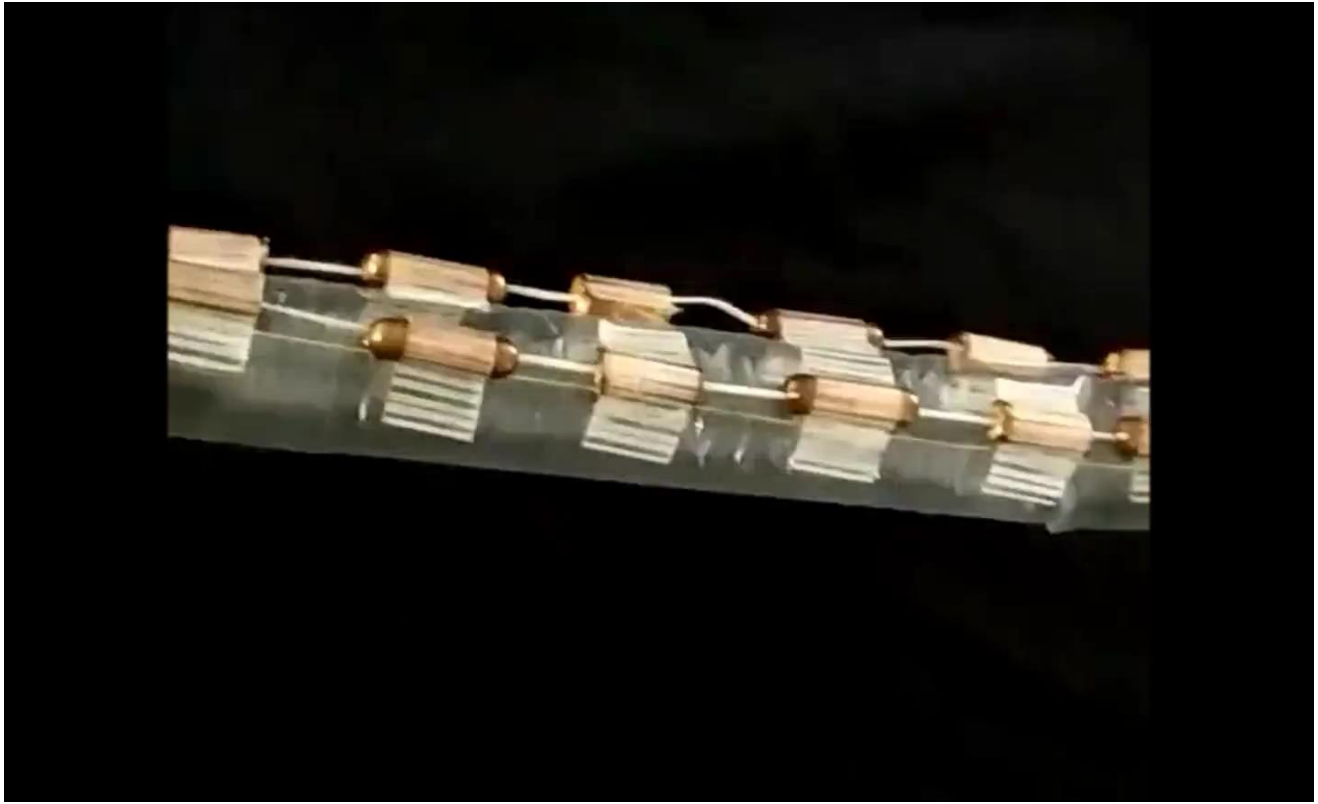
How do we achieve a desired shape of a growing robot through active actuation?





Designing Active Steering

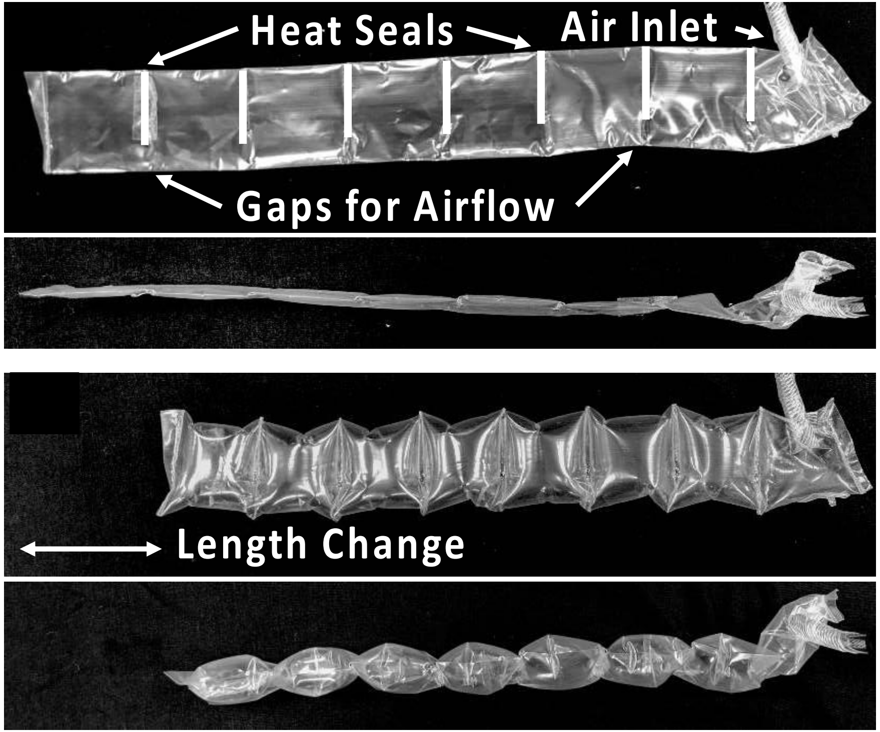
Tendon Actuation

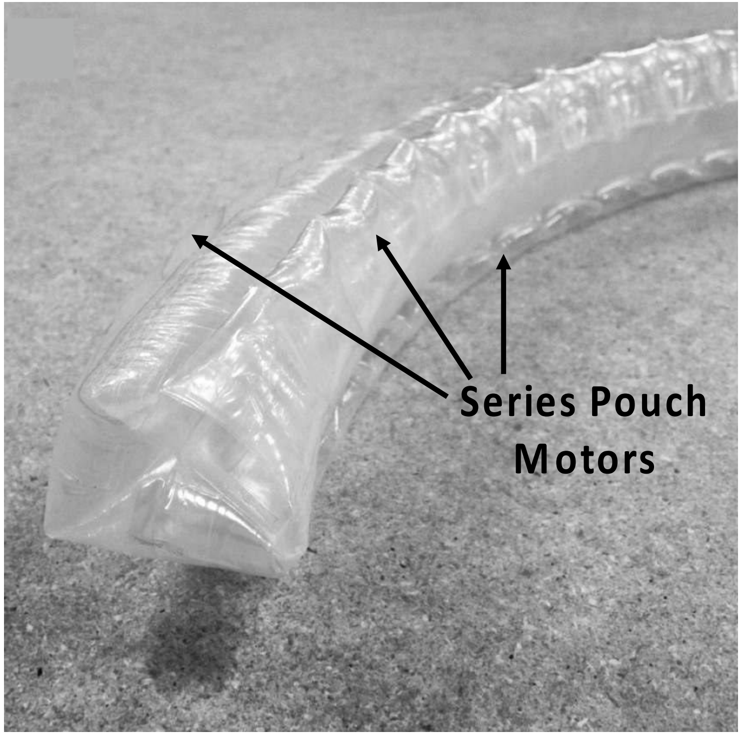


L. Gan, **L. H. Blumenschein**, Z. Huang, A. M. Okamura, E. W. Hawkes, and J. Fan (Accepted) 3D Electromagnetic Reconfiguration Enabled by Soft Continuum Robots. IEEE Robotics and Automation Letters, 2020.

Designing Active Steering

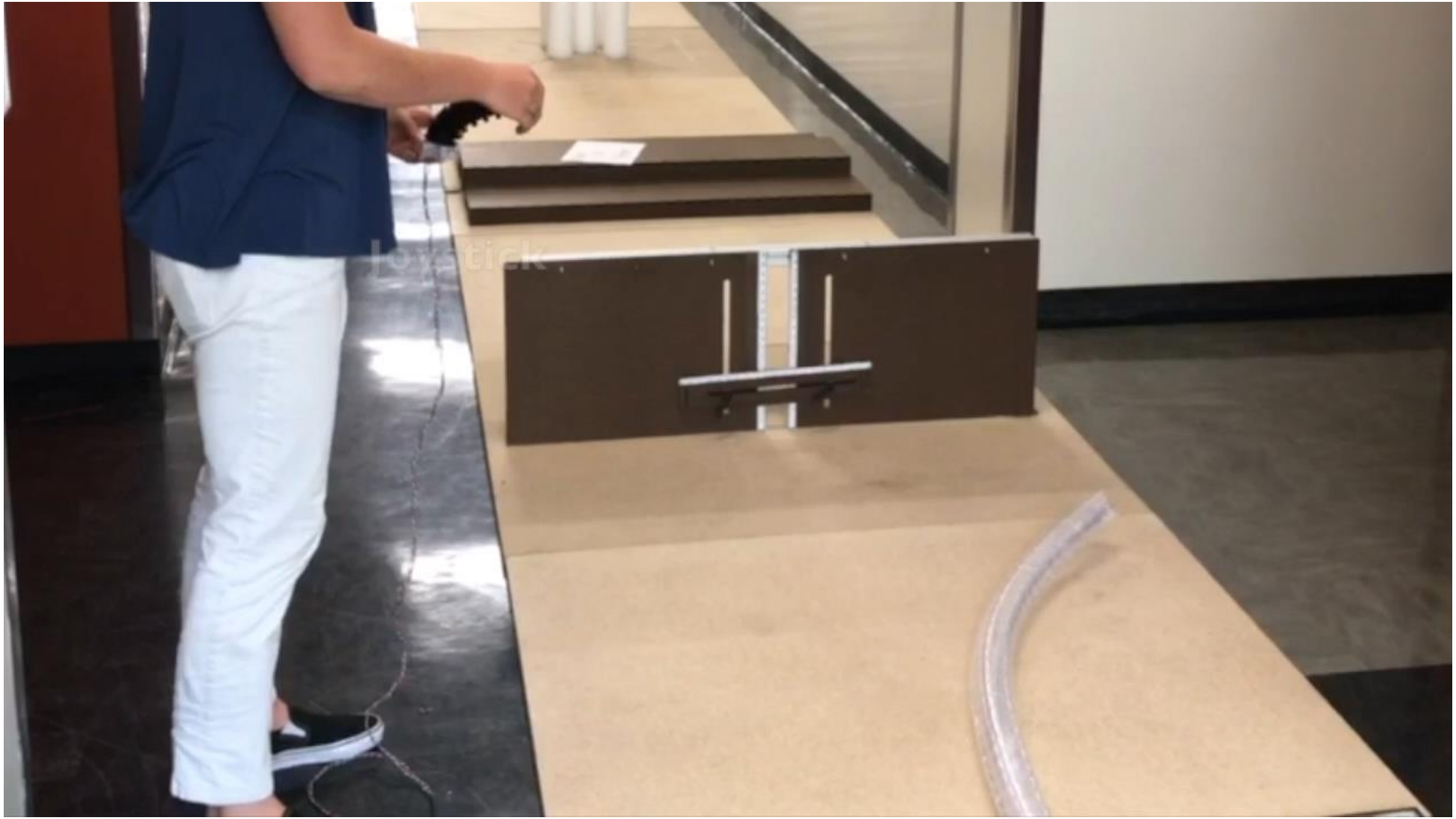
Pneumatic Actuation





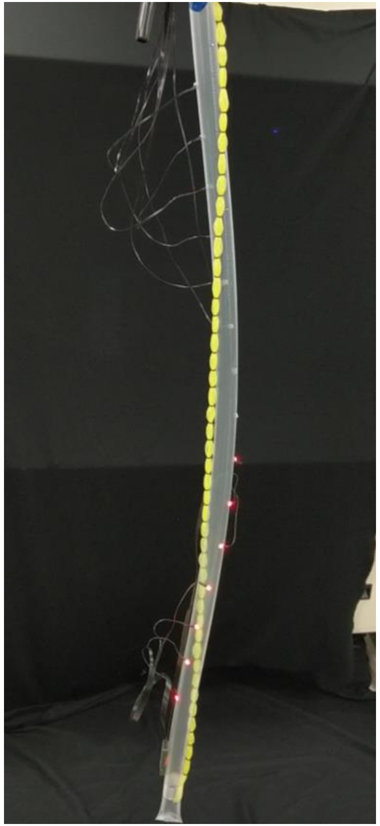
M. M. Coad, **L. H. Blumenschein**, S. Cutler, J. A. Reyna Zepeda, N. D. Naclerio, H. El-Hussieny, U. Mehmood, J.-H. Ryu, E. W. Hawkes, and A. M. Okamura (2020) *Vine Robots: Design, Teleoperation, and Deployment for Navigation and Exploration.* IEEE Robotics and Automation Magazine.

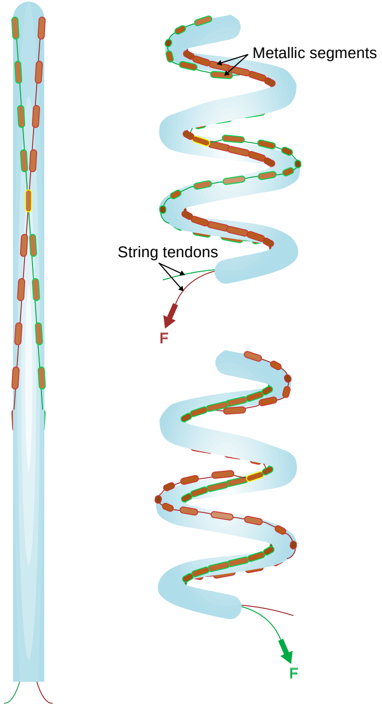
Designing Active Steering



J. D. Greer, T. K. Morimoto, A. M. Okamura, and E. W. Hawkes. Series Pneumatic Artificial Muscles (sPAMs) and Application to a Soft Continuum Robot. ICRA 2017. A Soft, Steerable Continuum Robot that Grows via Tip Extension. Soft Robotics, in press.

Creating More Complex Shapes

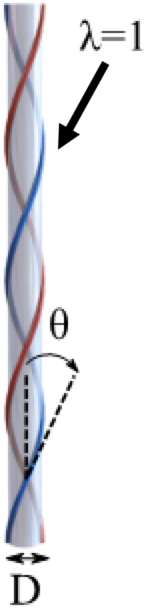




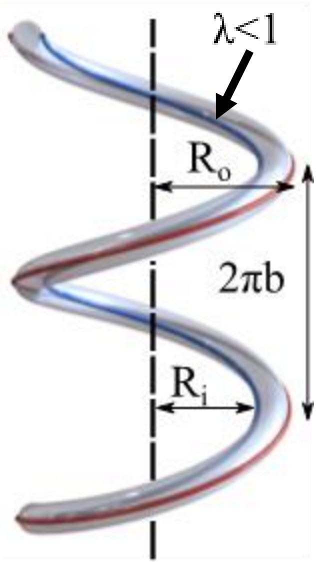
L. H. Blumenschein, N. S. Usevitch, B. Do, E. W. Hawkes, A. M. Okamura. Helical actuation on a soft inflated robot body. RoboSoft 2018.

General Actuator Kinematics

Uniform Actuation



Actuate blue path

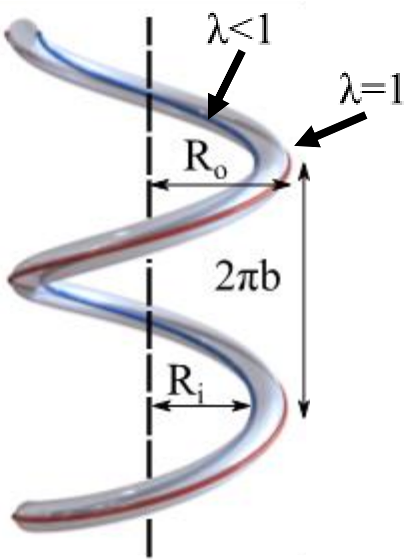


General Actuator Kinematics

Geometric Constraint: Path Length

Inner helix arc length is shortened relative to the outer helix:

$$\lambda = \frac{\sqrt{b^2 + {R_i}^2}}{\sqrt{b^2 + {R_o}^2}}$$



General Actuator Kinematics

Geometric Constraint: Cross-Sections

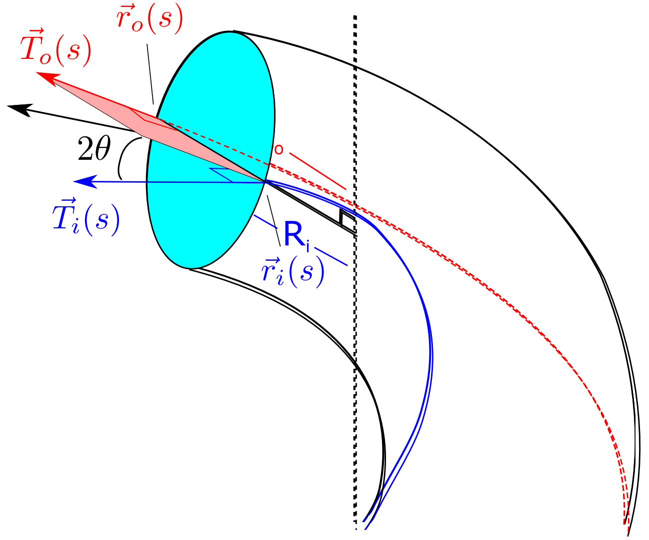
Tube diameter separates inner and outer helices:

$$D = R_o - R_i$$

Tangent vectors are offset by twice the actuator angle:

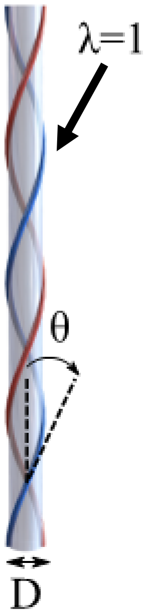
$$T_o(t) \cdot T_i(t) = \cos 2\theta$$

$$\cos 2\theta = \frac{b^2 + R_i R_o}{\sqrt{b^2 + {R_i}^2}\sqrt{b^2 + {R_o}^2}}$$



General Actuator Kinematics

Uniform Actuation Kinematics



Shape → Actuator

$$\boldsymbol{D} = R_o - R_i$$

$$\boldsymbol{\lambda} = \frac{\sqrt{b^2 + {R_i}^2}}{\sqrt{b^2 + {R_o}^2}}$$

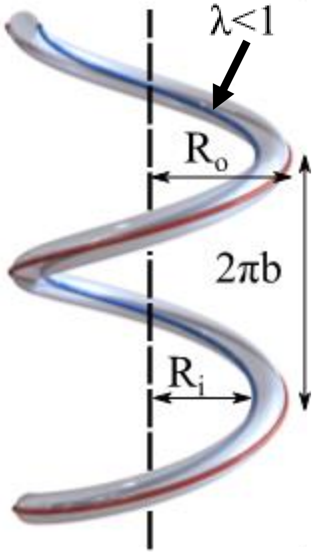
$$\boldsymbol{\theta} = \frac{1}{2}\cos^{-1}\frac{b^2 + R_i R_o}{\sqrt{b^2 + {R_i}^2}\sqrt{b^2 + {R_o}^2}}$$

Actuator → Shape

$$\boldsymbol{R_o} = \frac{D(1 - \lambda\cos 2\theta)}{1 - 2\lambda\cos 2\theta + \lambda^2}$$

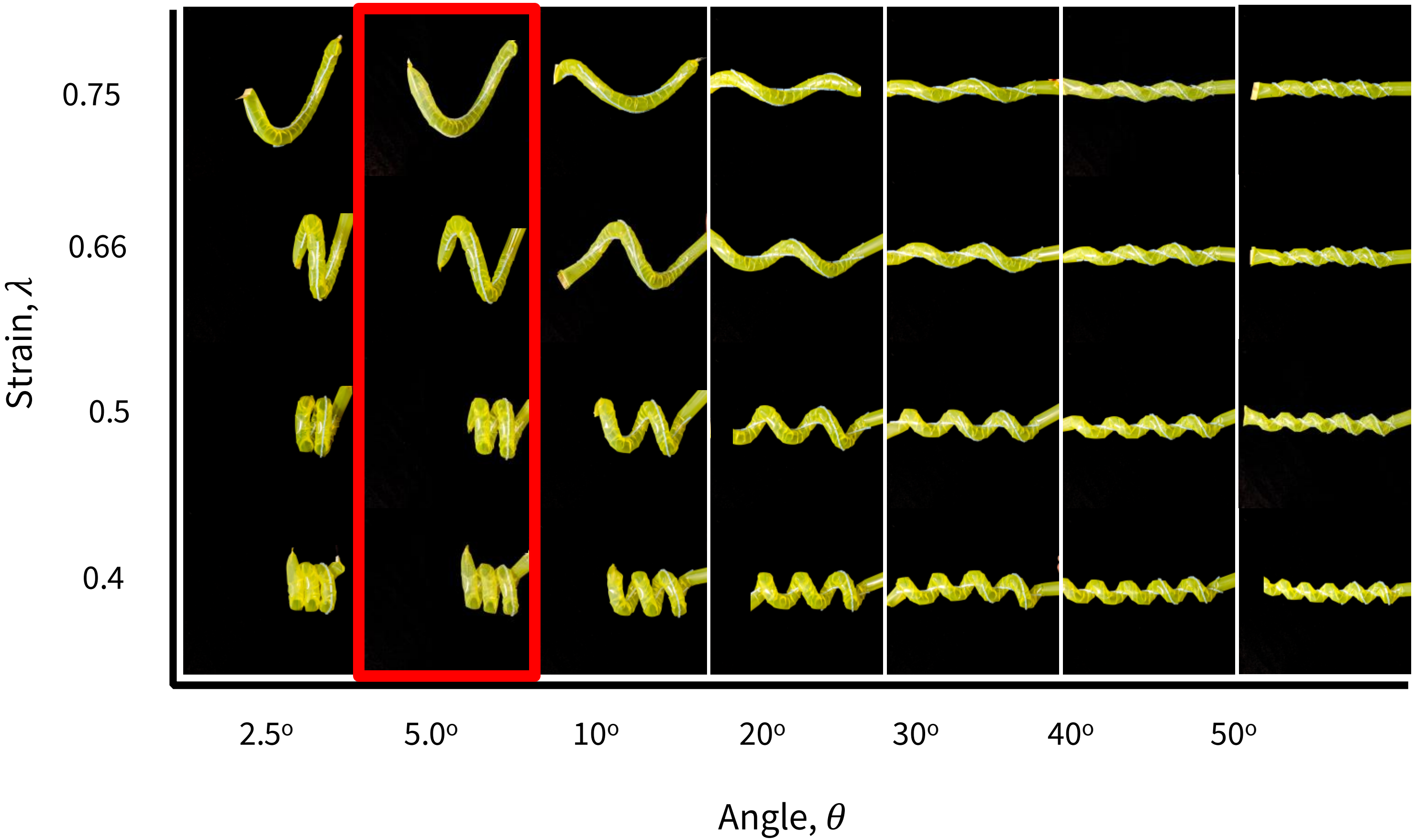
$$\boldsymbol{R_i} = \frac{D\lambda(\cos 2\theta - \lambda)}{1 - 2\lambda\cos 2\theta + \lambda^2}$$

$$\boldsymbol{b} = \frac{D\lambda\sin 2\theta}{1 - 2\lambda\cos 2\theta + \lambda^2}$$



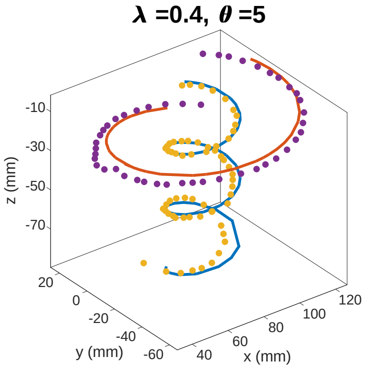
General Actuator Kinematics

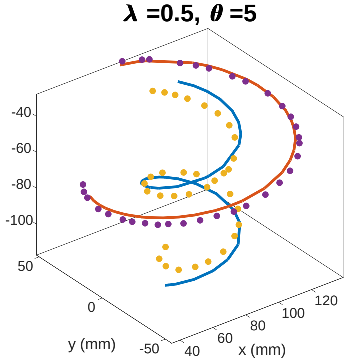
Model Validation: Helices

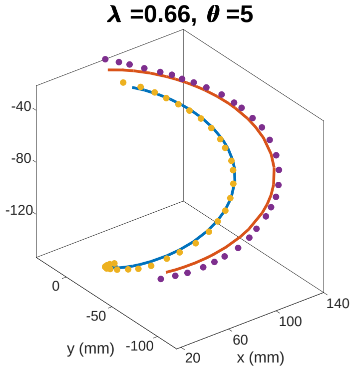


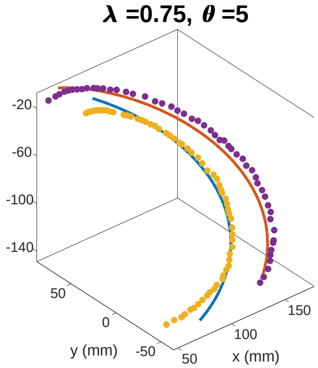
General Actuator Kinematics

Model Validation: Helices







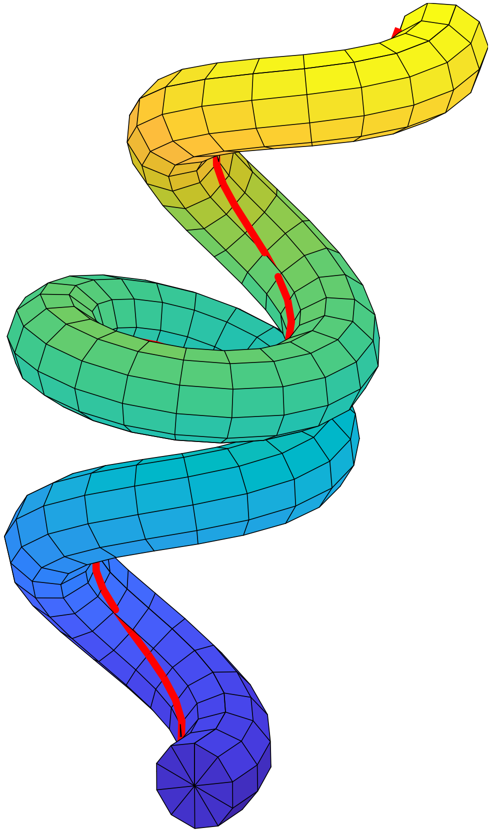


Inner Model   Outer Model   Inner Measured   Outer Measured

General Actuator Kinematics

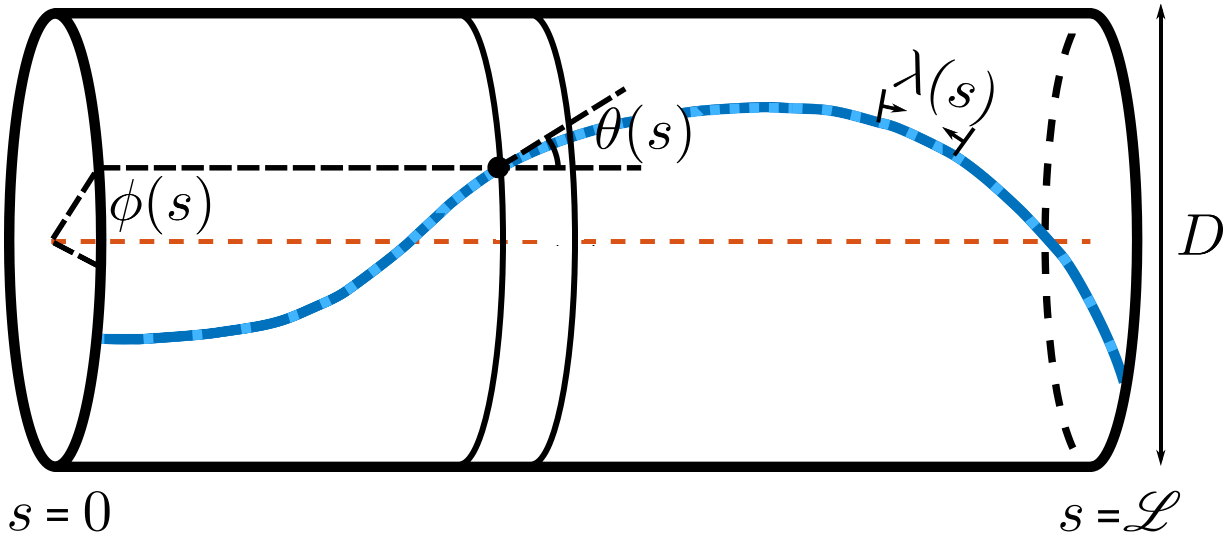
Generalizing Beyond Helices

$\lambda = 0.5$
$D = 0.5$
$\theta: 10^o \rightarrow 5^o \rightarrow 10^o$



General Actuator Kinematics

Generalizing Beyond Helices



General Actuator Kinematics

Generalizing Beyond Helices

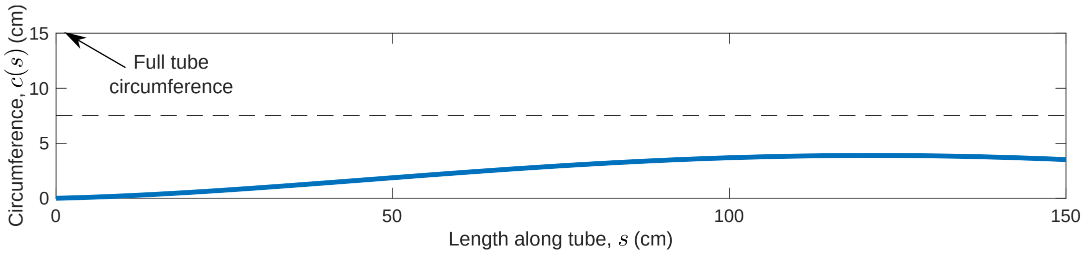
Transformation along a helical segment:

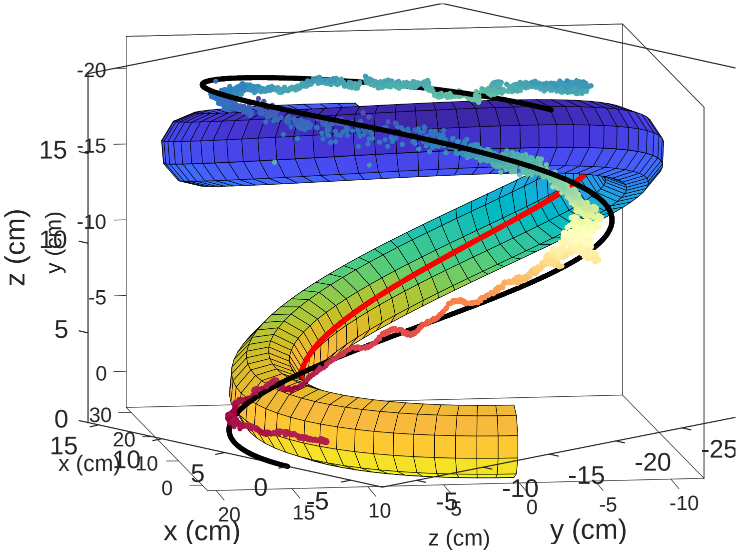
$$T_c(s) = \begin{bmatrix} \frac{R^2}{L^2}\cos\frac{\Delta\ell_\lambda}{L} + \frac{b^2}{L^2} & \frac{-R}{L}\sin\frac{\Delta\ell_\lambda}{L} & \frac{Rb}{L^2}\left(1 - \cos\frac{\Delta\ell_\lambda}{L}\right) & \frac{R^2}{L}\sin\frac{\Delta\ell_\lambda}{L} + \frac{b^2}{L}\frac{\Delta\ell_\lambda}{L} \\ \frac{R}{L}\sin\frac{\Delta\ell_\lambda}{L} & \cos\frac{\Delta\ell_\lambda}{L} & \frac{-b}{L}\sin\frac{\Delta\ell_\lambda}{L} & R\left(1 - \sin\frac{\Delta\ell_\lambda}{L}\right) \\ \frac{Rb}{L^2}\left(1 - \cos\frac{\Delta\ell_\lambda}{L}\right) & \frac{b}{L}\sin\frac{\Delta\ell_\lambda}{L} & \frac{b^2}{L^2}\cos\frac{\Delta\ell_\lambda}{L} + \frac{R^2}{L^2} & \frac{Rb}{L}\left(\frac{\Delta\ell_\lambda}{L} - \sin\frac{\Delta\ell_\lambda}{L}\right) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $L = \sqrt{R^2 + b^2}$ and $\Delta\ell_\lambda = \Delta\ell\sqrt{\frac{\lambda^2 + 2\lambda\cos 2\theta + 1}{2(1 + \cos 2\theta)}}$ (along the centerline)

General Actuator Kinematics

Model Validation: Static Shapes

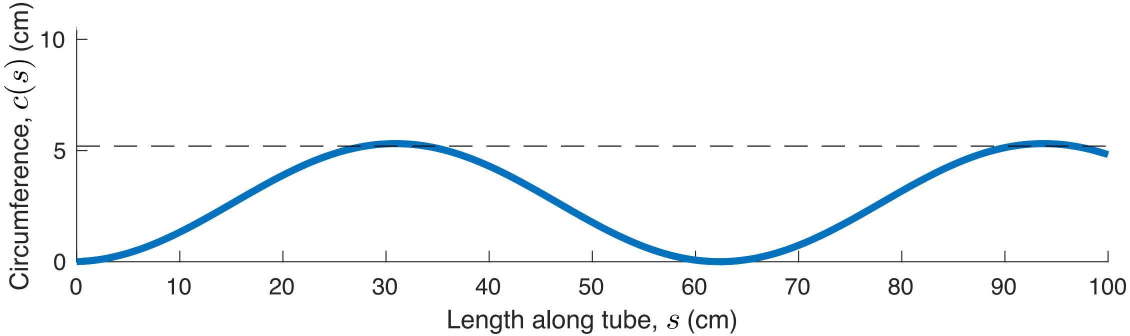


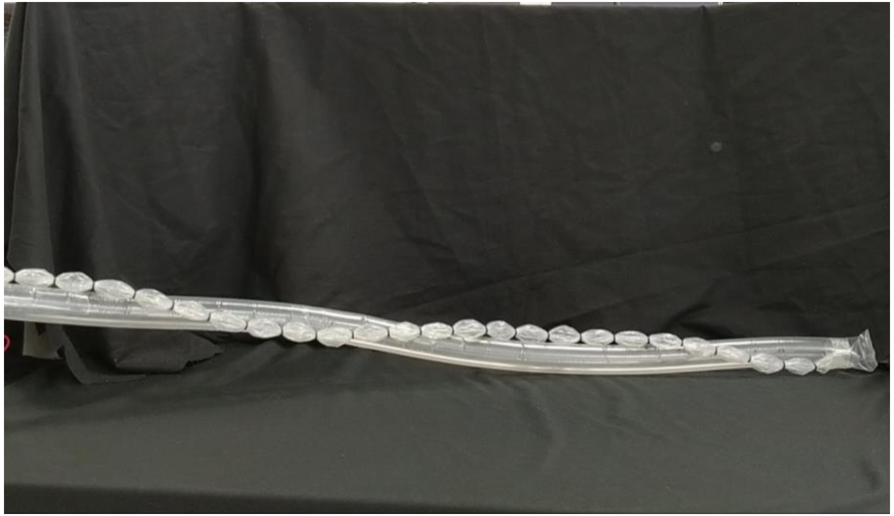


RMSE = 0.45 cm

General Actuator Kinematics

Model Validation: Pneumatic Actuation





General Actuator Kinematics

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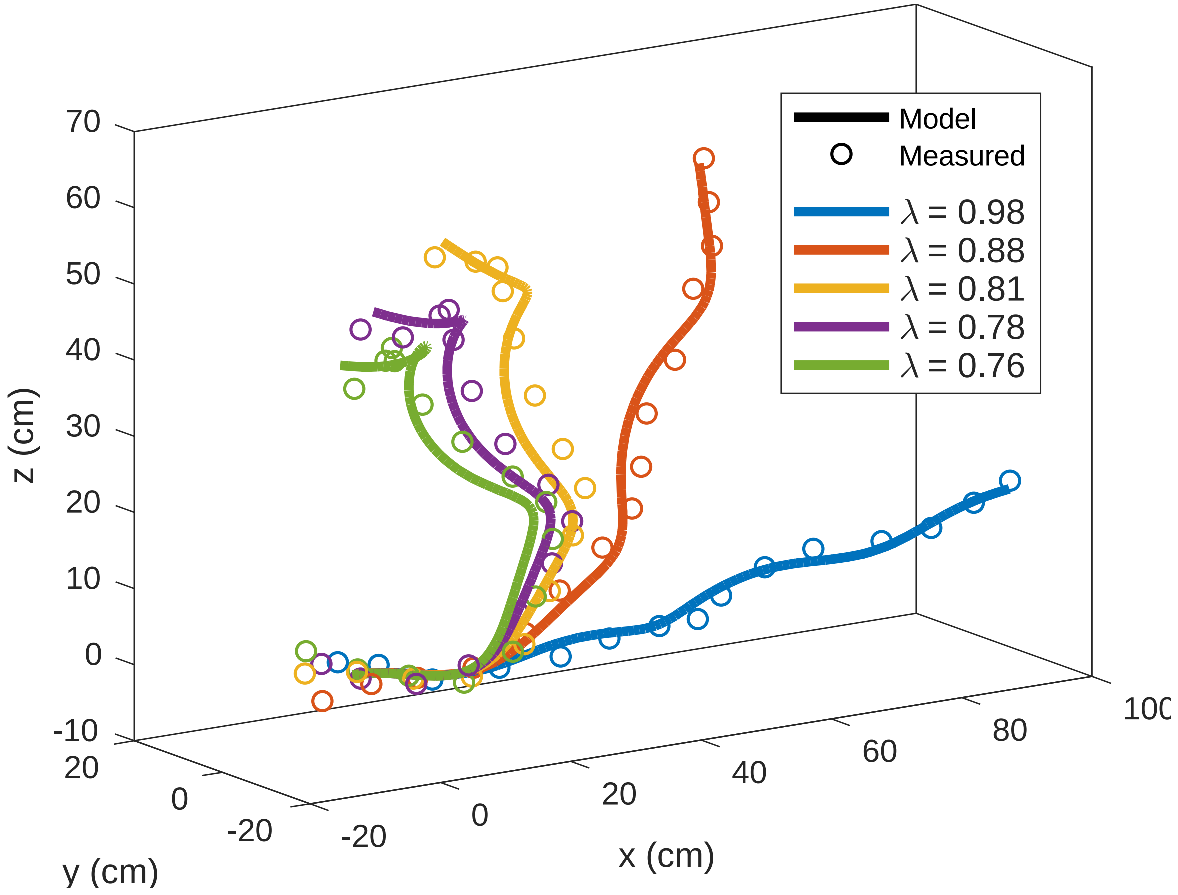


General Actuator Kinematics

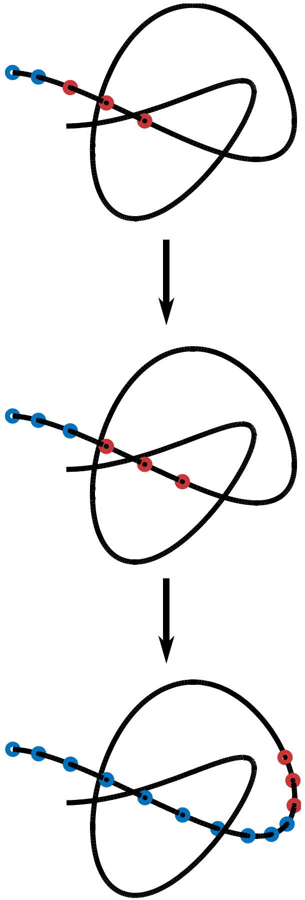
Shape Matching Algorithm

- Find $\boldsymbol{\theta}, \boldsymbol{\lambda}$ fit for each section of target shape to minimize error

- Consider the best fit for next $\boldsymbol{n}$ unfit sections

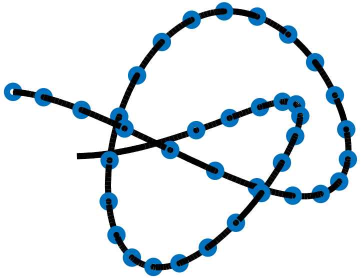
- Save the $\boldsymbol{\theta}, \boldsymbol{\lambda}$ for the next segment only

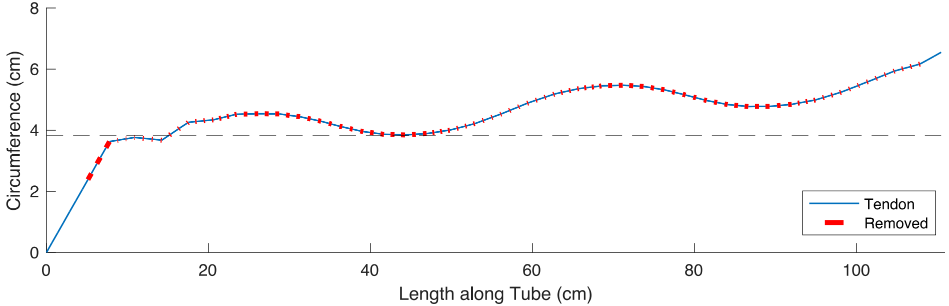
- Repeat for length of target shape



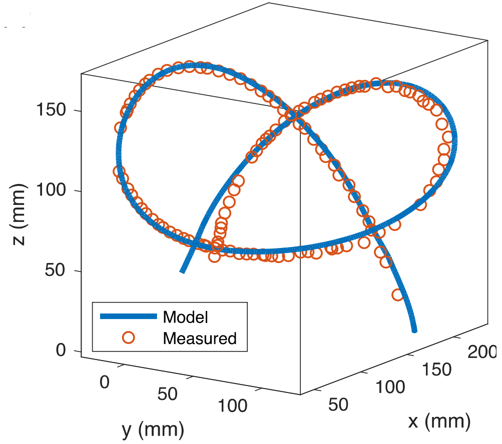
General Actuator Kinematics

Matching Desired Shapes





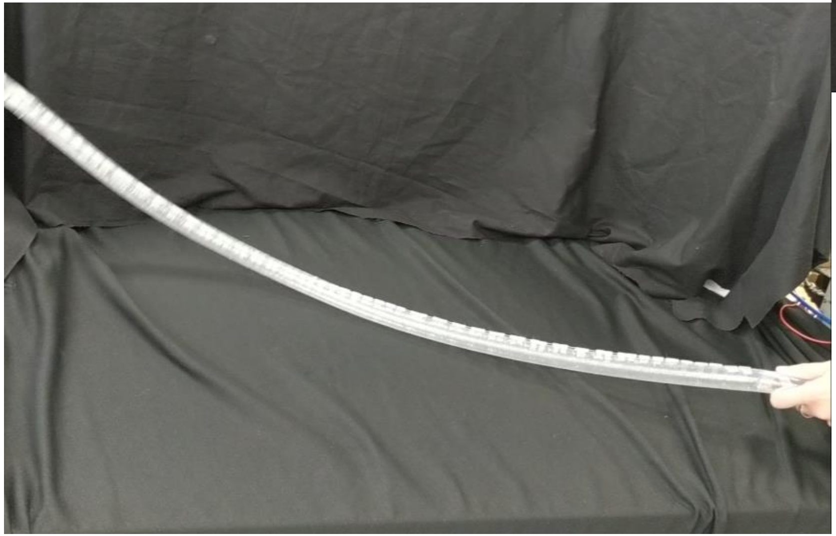
RMSE = 6.88 mm



General Actuator Kinematics

Growth and Actuation

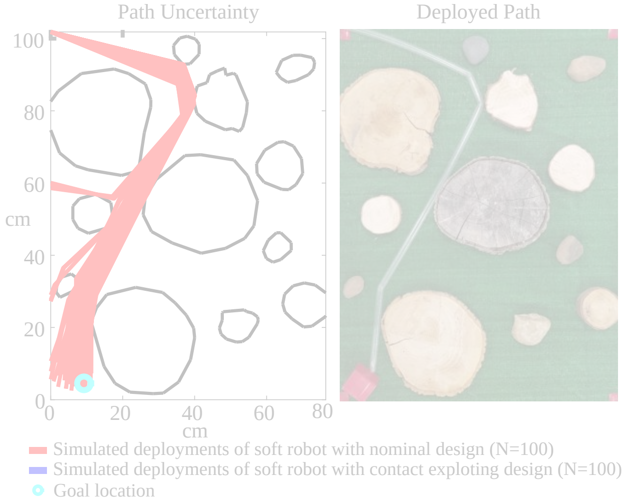
During growth →



← After growth

Applying geometric constraints to simplify kinematic models

Obstacle interaction to decrease uncertainty



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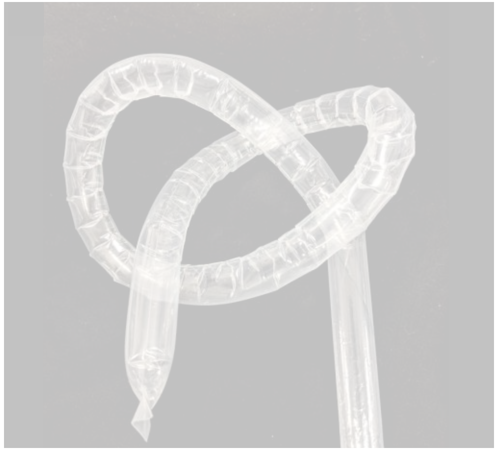

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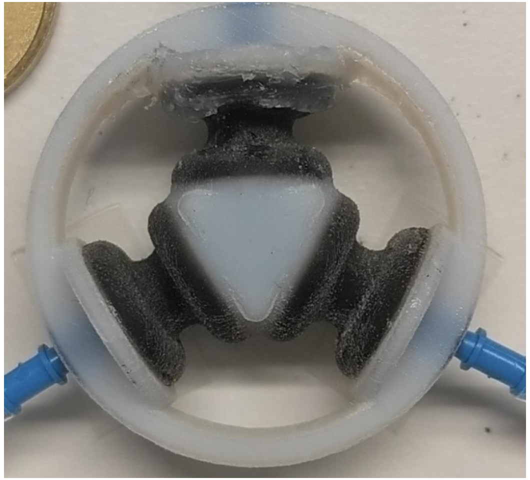

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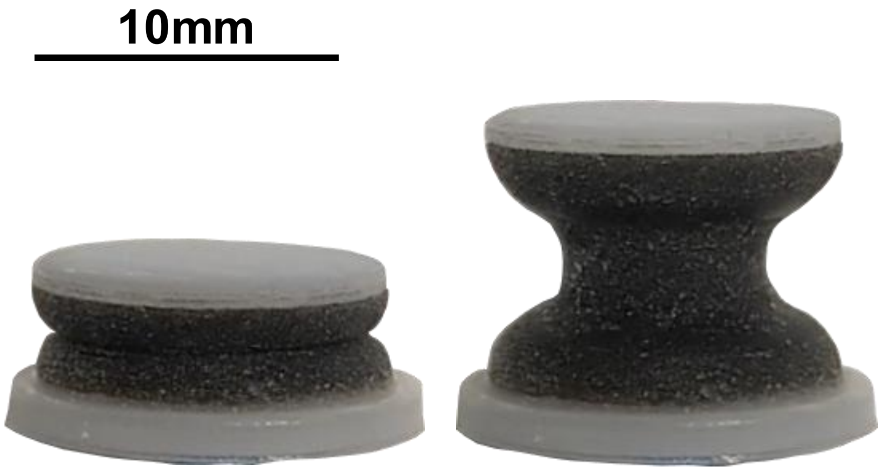
Component Actuator: Soft Bellows

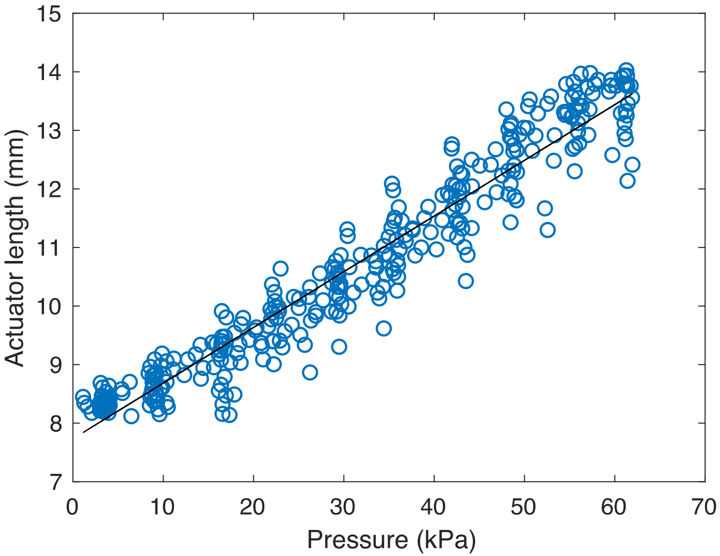
Multi-material polyjet printing (Agilus and Digital ABS)

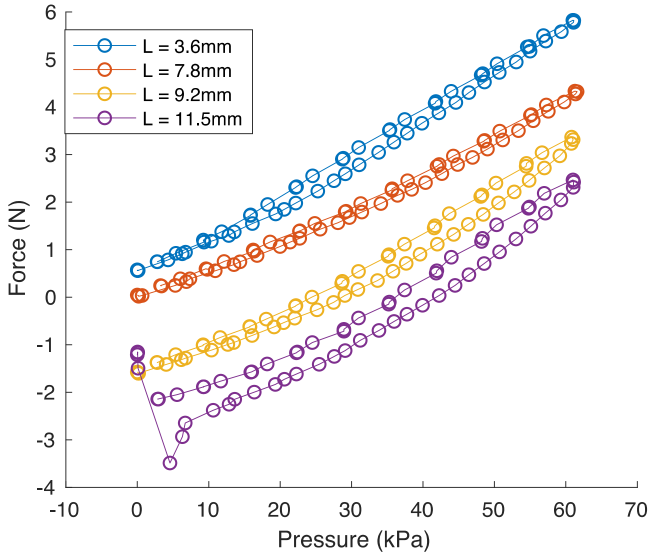
Change length through bending (and stretching) wall material

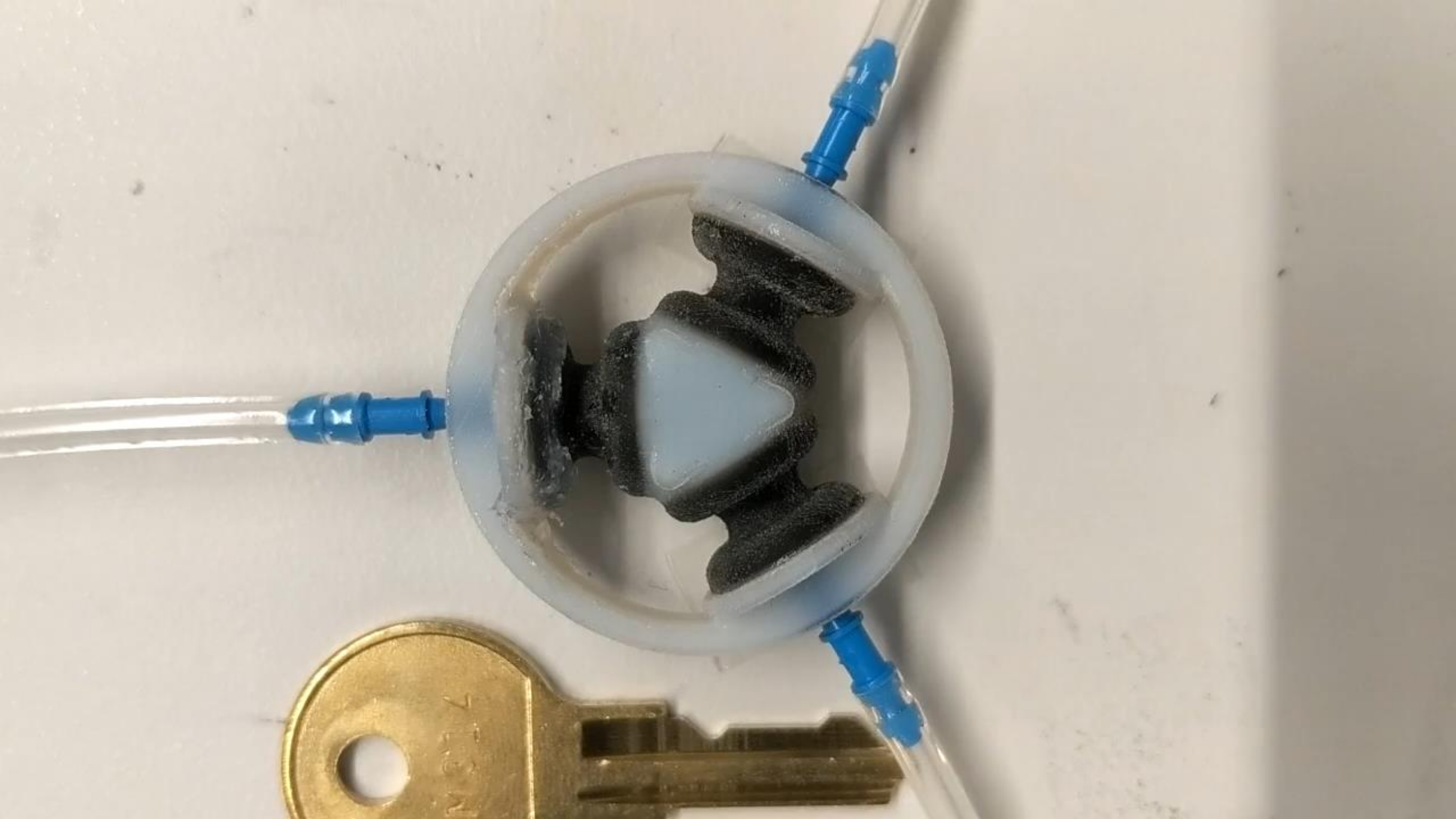
Total length change 340%



Component Actuator: Soft Bellows





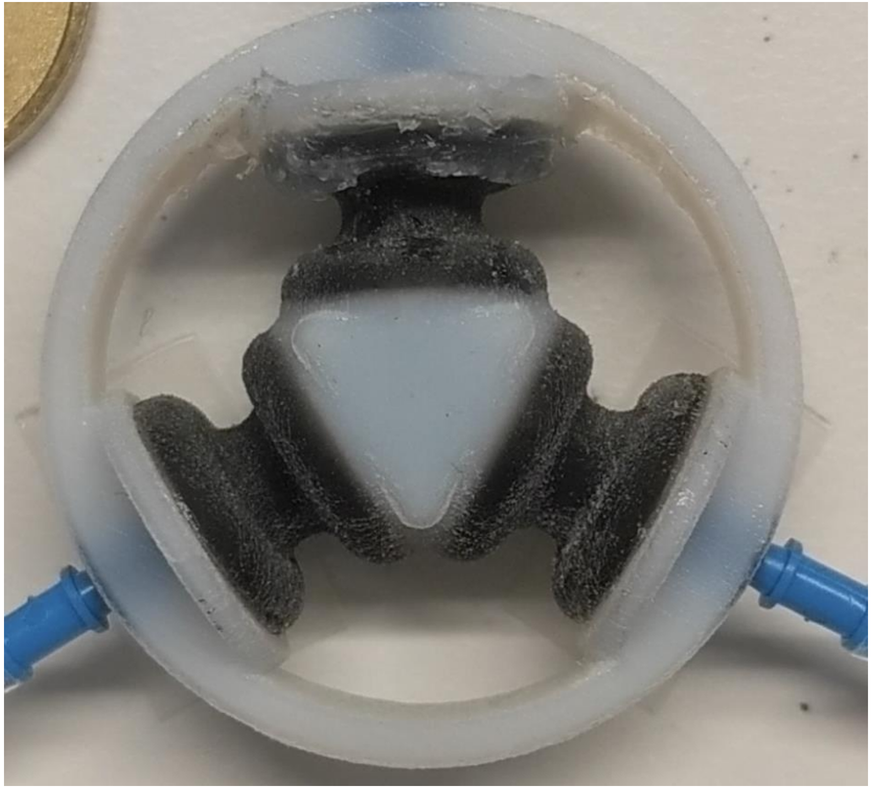


Jacobian Model

$$\vec{F}_o = \mathbf{J} * \vec{P} = \sum_{i=1}^{n} P_i A_i \hat{u}_i$$

$$\mathbf{J} = A[\hat{u_1}\hat{u_2}...\hat{u_n}]$$

$$\hat{u}_i = \begin{bmatrix} \cos(\frac{2\pi}{n}(i-1)+\theta_o) \\ \sin(\frac{2\pi}{n}(i-1)+\theta_o) \end{bmatrix}$$



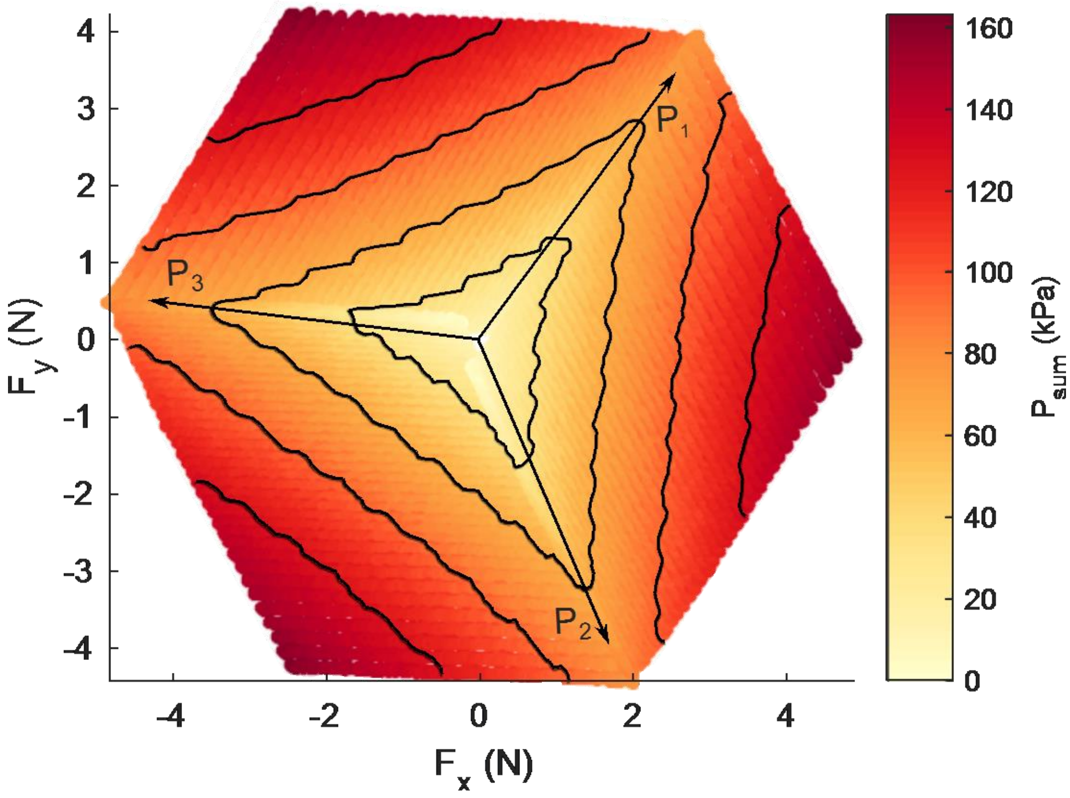
Force Workspace

$$\vec{F_o} = \mathbf{J} * \vec{P} = \sum_{i=1}^{n} P_i A_i \hat{u_i}$$

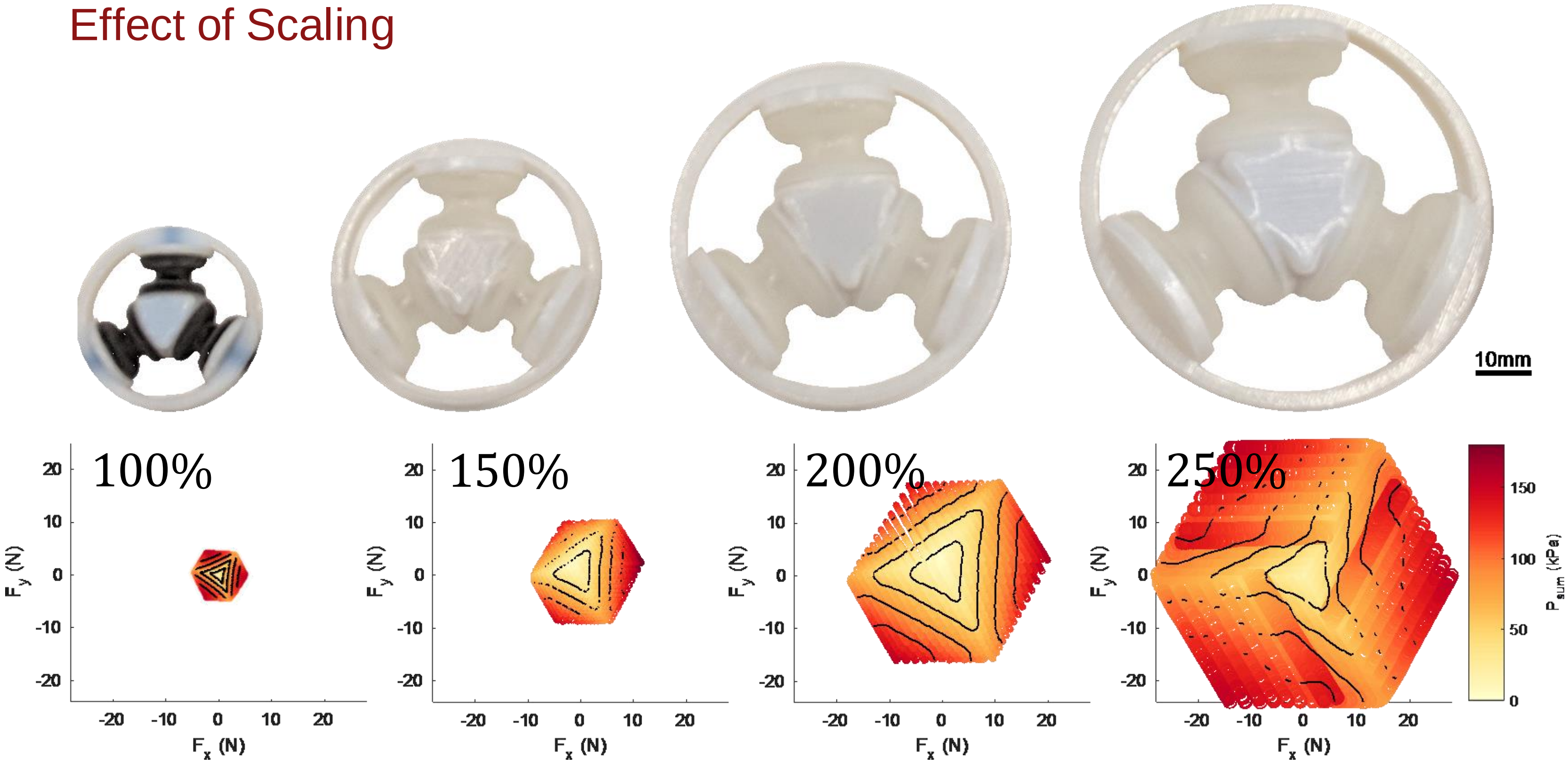
$$\mathbf{J} = A[\hat{u_1}\hat{u_2}...\hat{u_n}];$$

$$\hat{u_i} = \begin{bmatrix} \cos(\frac{2\pi}{n}(i-1)+\theta_o) \\ \sin(\frac{2\pi}{n}(i-1)+\theta_o) \end{bmatrix}$$

$A = 54.4\text{mm}^2; \theta_o = -56^o$
$R^2 = 0.964$

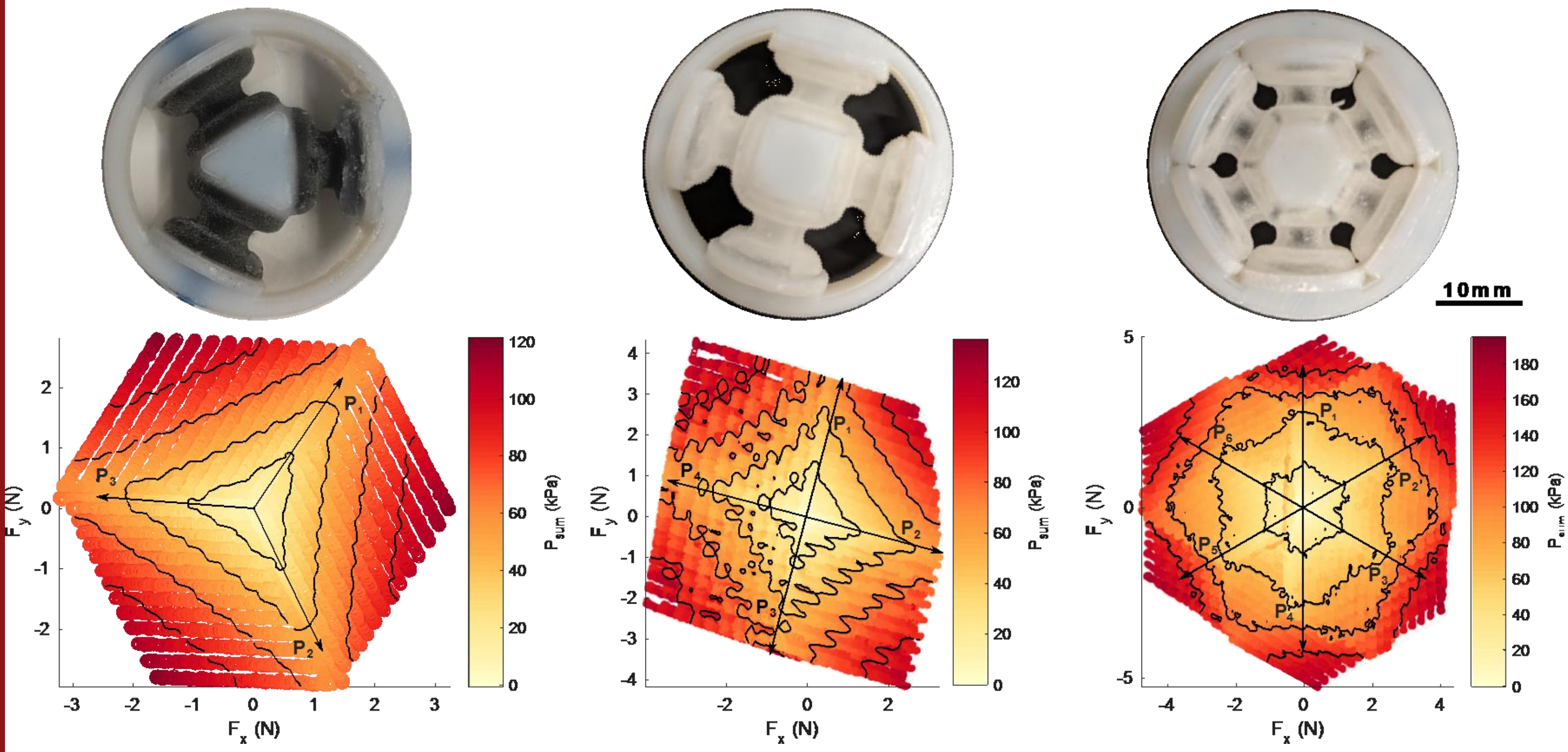


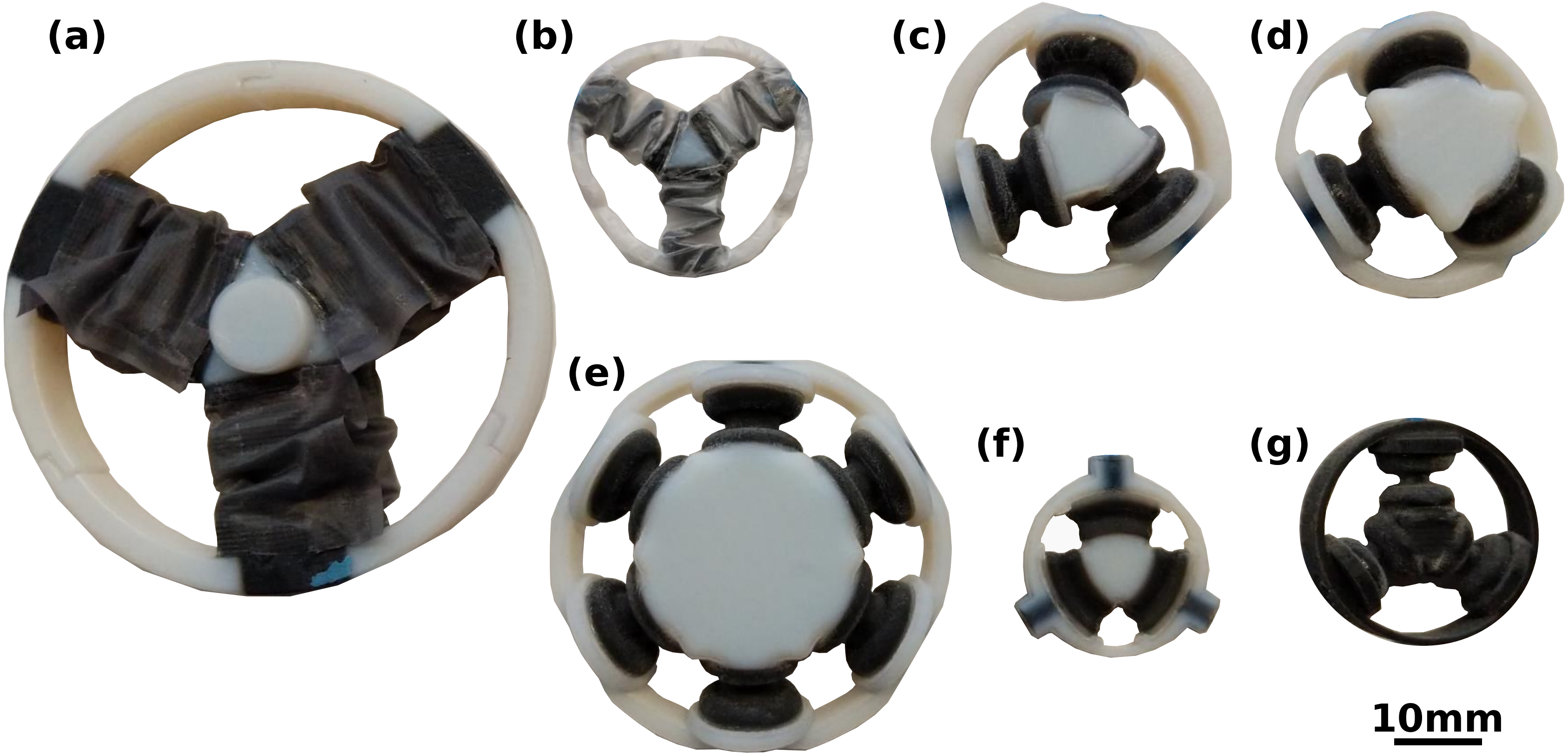
Effect of Scaling



$A = 54.4\ \mathrm{mm}^2$	$A = 126.3\ \mathrm{mm}^2$	$A = 237.3\ \mathrm{mm}^2$	$A = 305.4\ \mathrm{mm}^2$
$F_{max} = 4.9\ \mathrm{N}$	$F_{max} = 12.1\ \mathrm{N}$	$F_{max} = 20.8\ \mathrm{N}$	$F_{max} = 27.8\ \mathrm{N}$
	155%	208%	238%

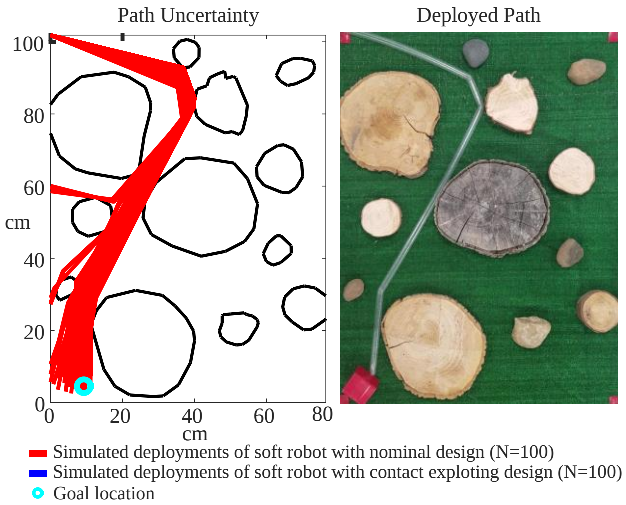
Increasing Component Actuators




(a)
(b)
(c)
(d)
(e)
(f)
(g)
10mm

Applying geometric constraints to simplify kinematic models

Obstacle interaction to decrease uncertainty

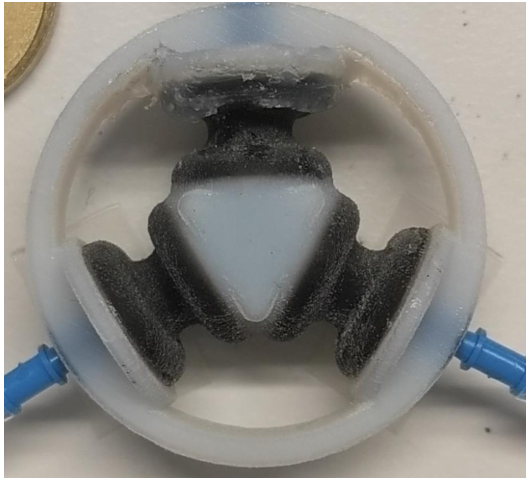


Greer, et al (2020). “Robust navigation of a soft growing robot by exploiting contact with the environment,” *IJRR*.

Geometric models of general actuation

Blumenschein, et al (2020). “Geometric Solutions for General Actuator Routing on Inflated-Beam Soft Growing Robots,” *arXiv preprint arXiv:2006.06117*.

Design of soft delta mechanisms



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Conclusions

- Can develop building blocks for design and modeling through observation of heuristics, decomposition of complex designs, or targeted design.
- Created models predict behavior well enough to design more complex interactions
- The overall accuracy is limited by the assumptions and simplifications made when applying the geometric constraints
- In the future, applying methods like these can lead to more rapid prototyping and understanding of new soft robotic functions

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Questions?