Leveraging Data and the Koopman Operator to Build Control-oriented Models of Soft Robots

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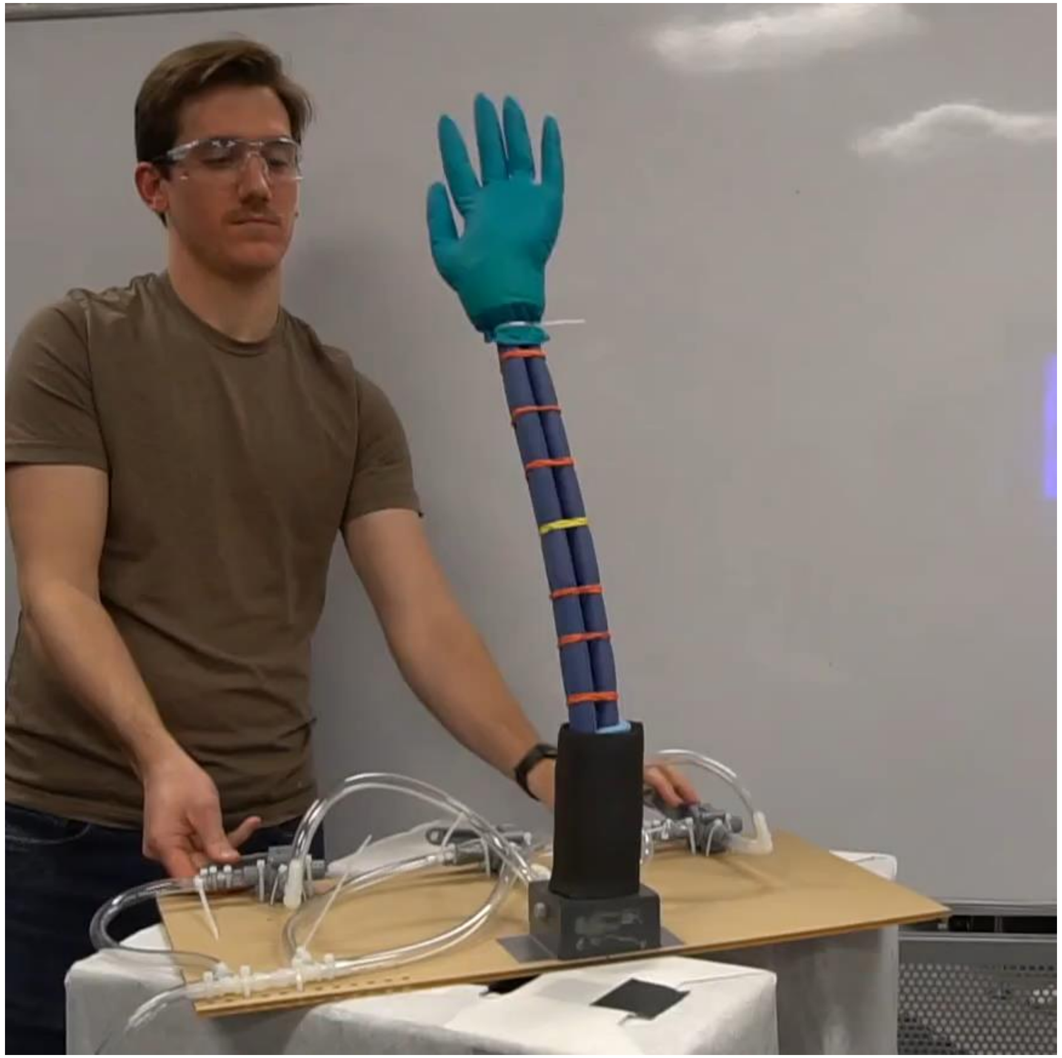
Modeling Soft Robots: Capabilities and Limitations Workshop

Soft robots are well suited for data-driven modeling methods

- Safe to collect data
- Avoids simplifying assumptions
- Not robot specific



Koopman-based modeling approach yields control-oriented models

- Accurate global linear representation of dynamics
- Enables real-time control
- Accommodates loading conditions

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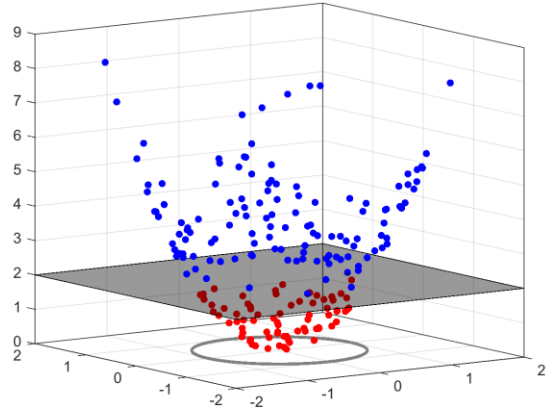
Lifting data can yield a more useful representation

Lifted Space

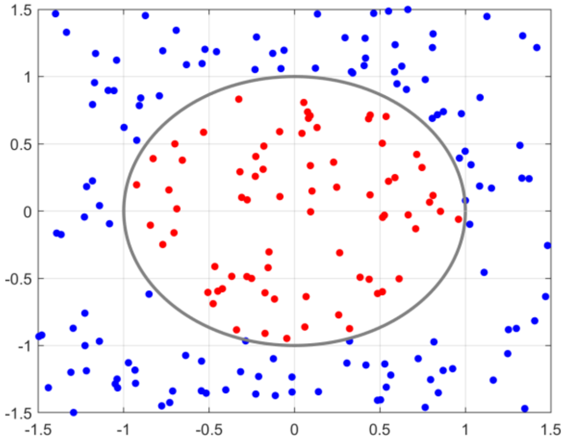
Classification



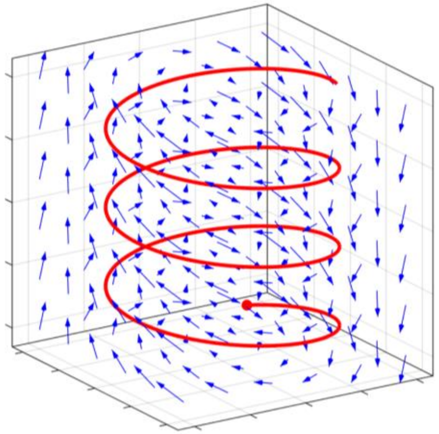
Lift

$$\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \\ x^2 + y^2 \end{bmatrix}$$

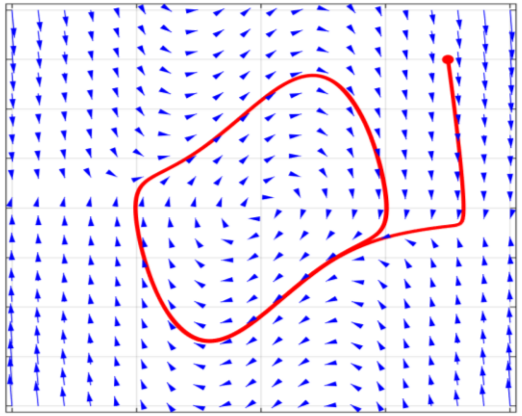
Original Space



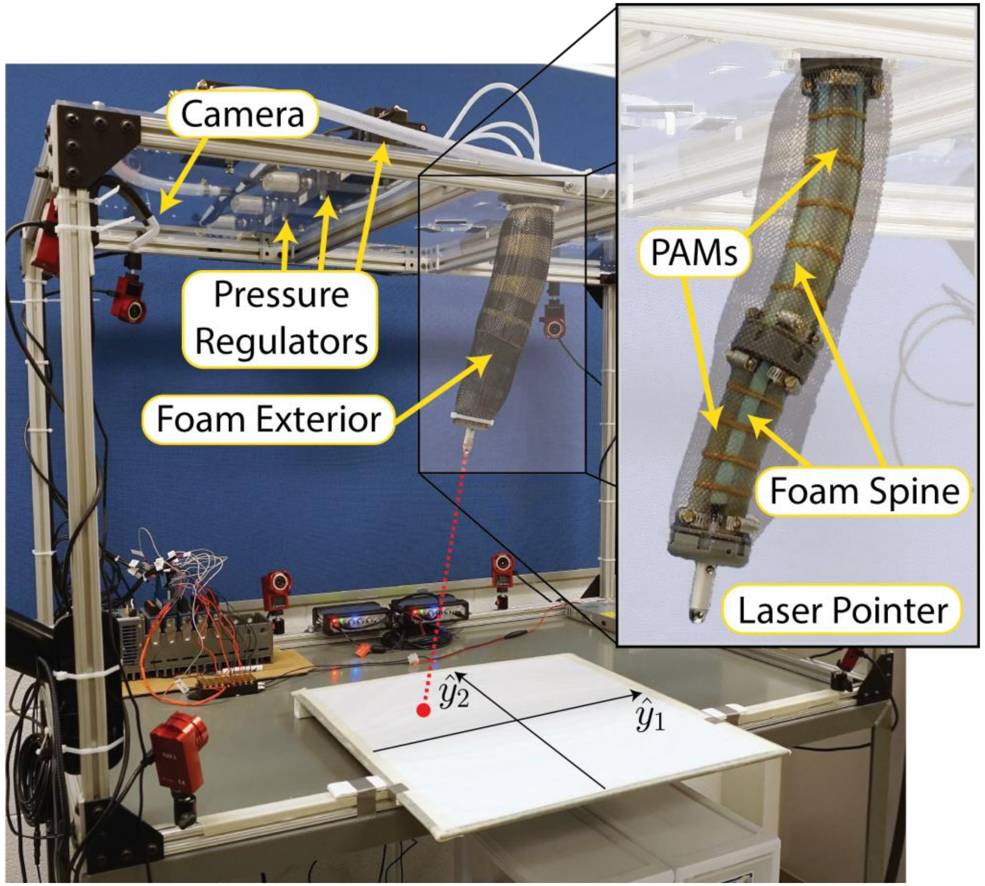
Dynamics

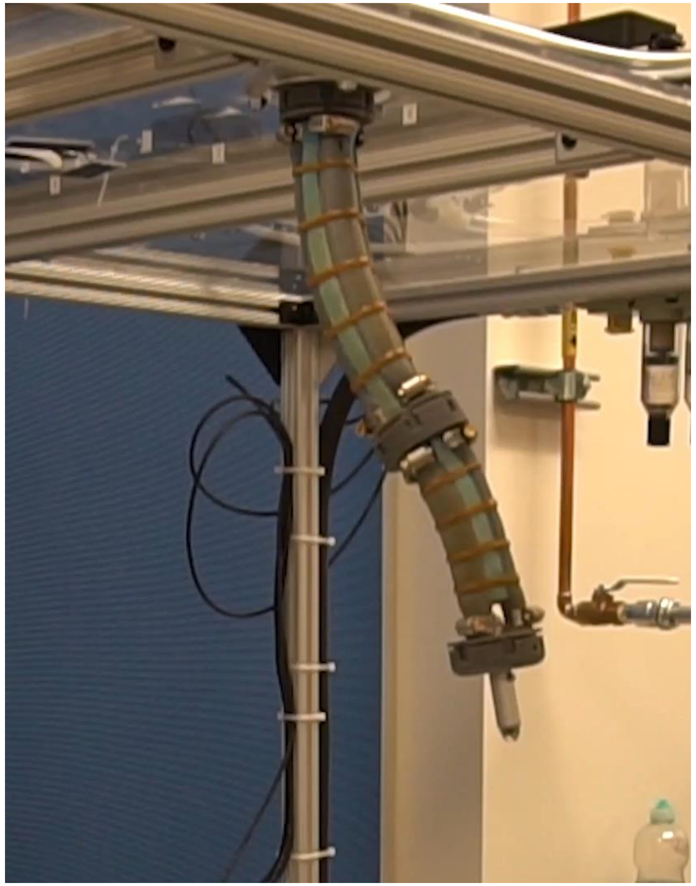


Lift



Koopman modeling approach was applied to a soft robot arm





Input: Pressure regulator voltages (3D)
State: Laser dot coordinates (2D)

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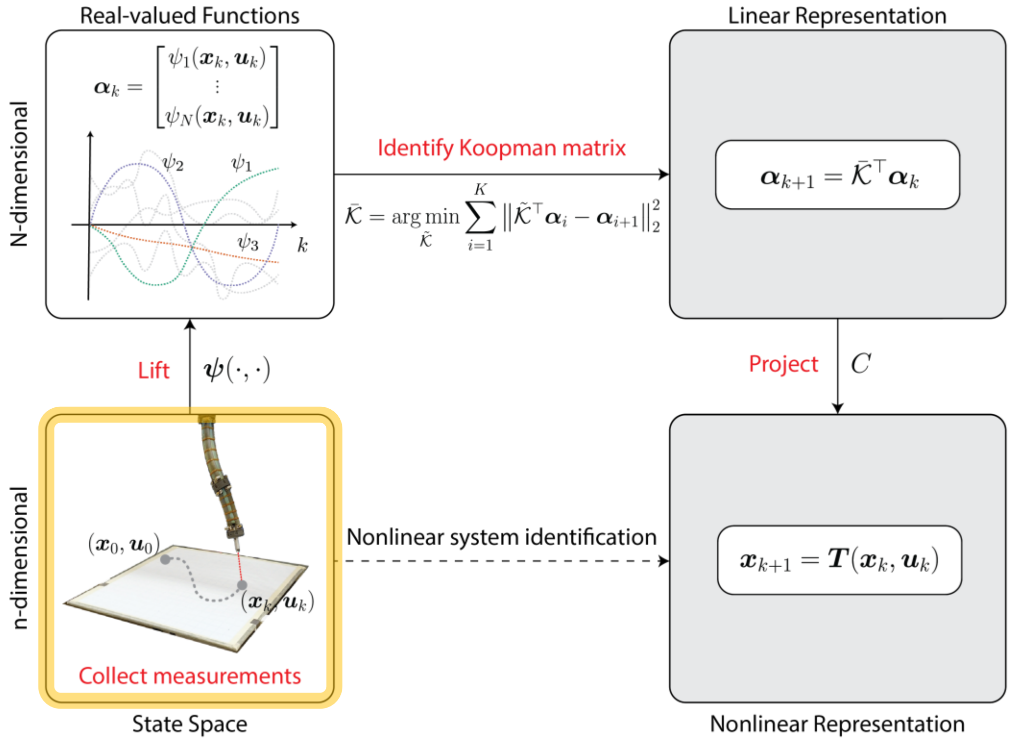
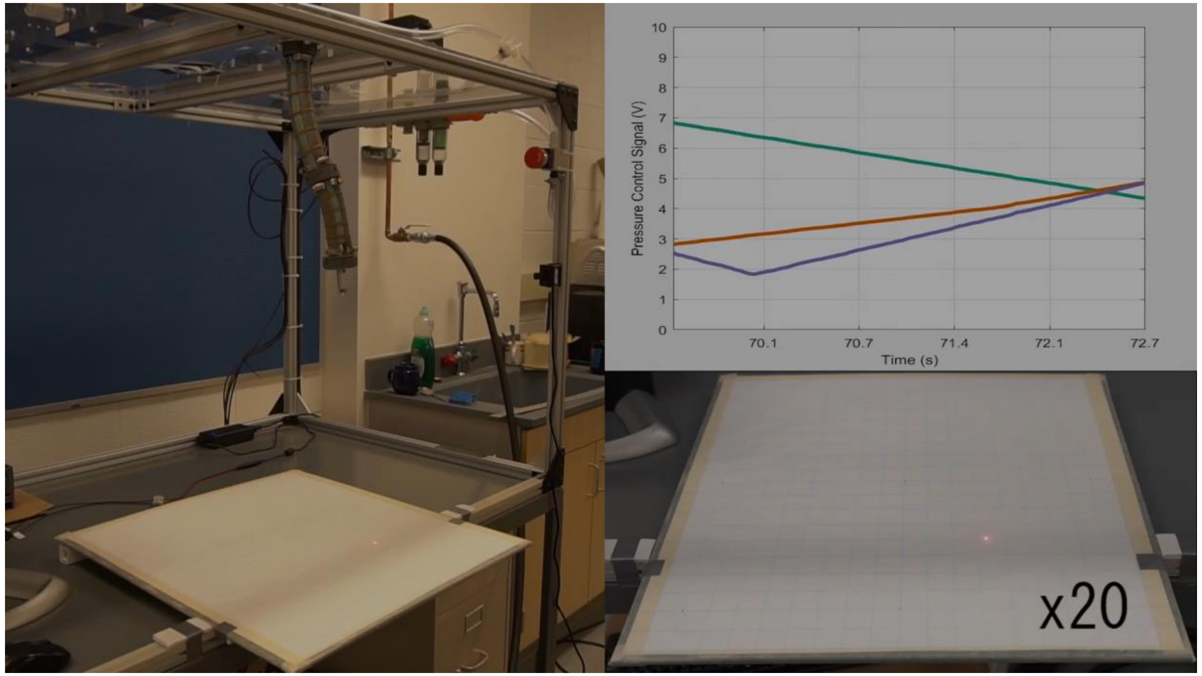
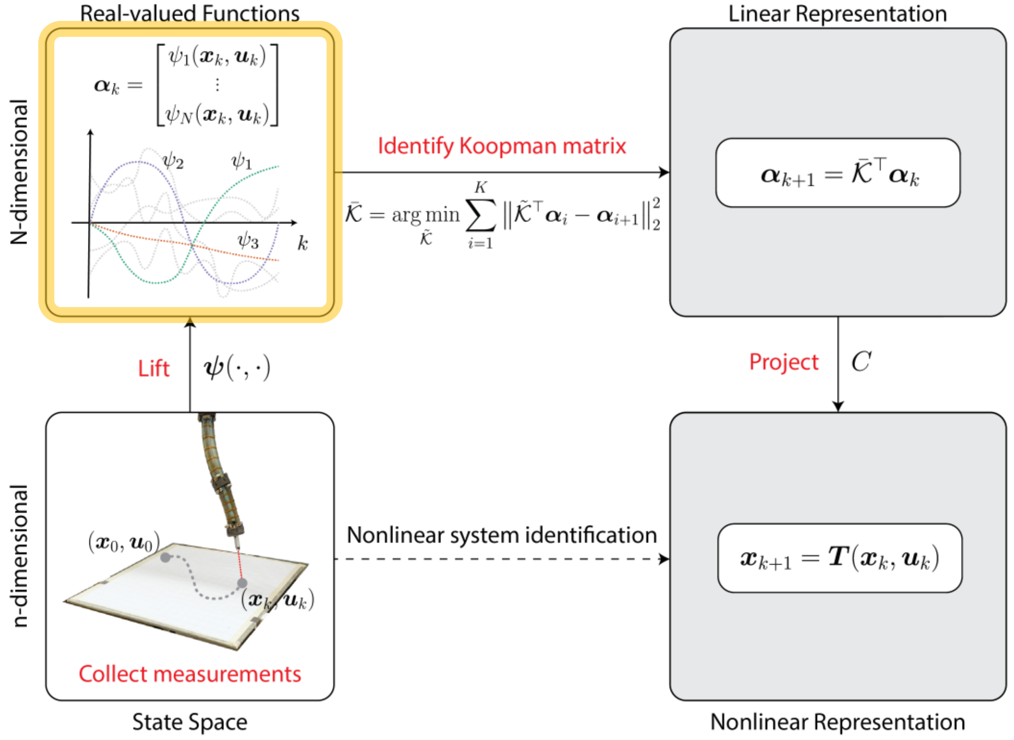
Data is collected under random inputs



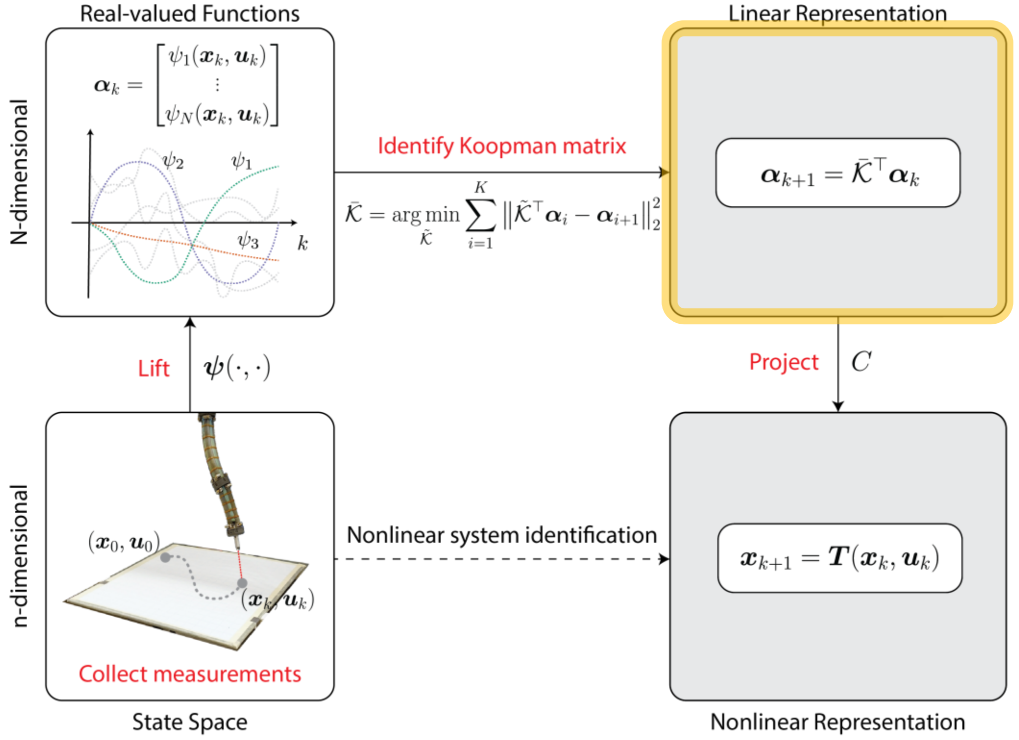


Data is lifted using polynomial basis functions



$$\boldsymbol{\psi}(\boldsymbol{x}, \boldsymbol{u}) = \left.\begin{bmatrix} \boldsymbol{x} \\ x_{[1]}^2 \\ x_{[1]} x_{[2]} \\ x_{[2]}^2 \\ \vdots \\ \boldsymbol{u} \end{bmatrix}\right\} \boldsymbol{z}(\boldsymbol{x}) \longleftarrow \text{“lifted state”}$$

Koopman matrix is identified via linear regression



$$\bar{\mathcal{K}}^\top = \begin{bmatrix} 0.973 & 0.057 & -0.034 & -0.050 & \cdots \\ 0.091 & 0.812 & -0.081 & 0.120 & \cdots \\ 1.000 & 0.000 & -0.000 & -0.000 & \cdots \\ -0.000 & 1.000 & 0.000 & 0.000 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

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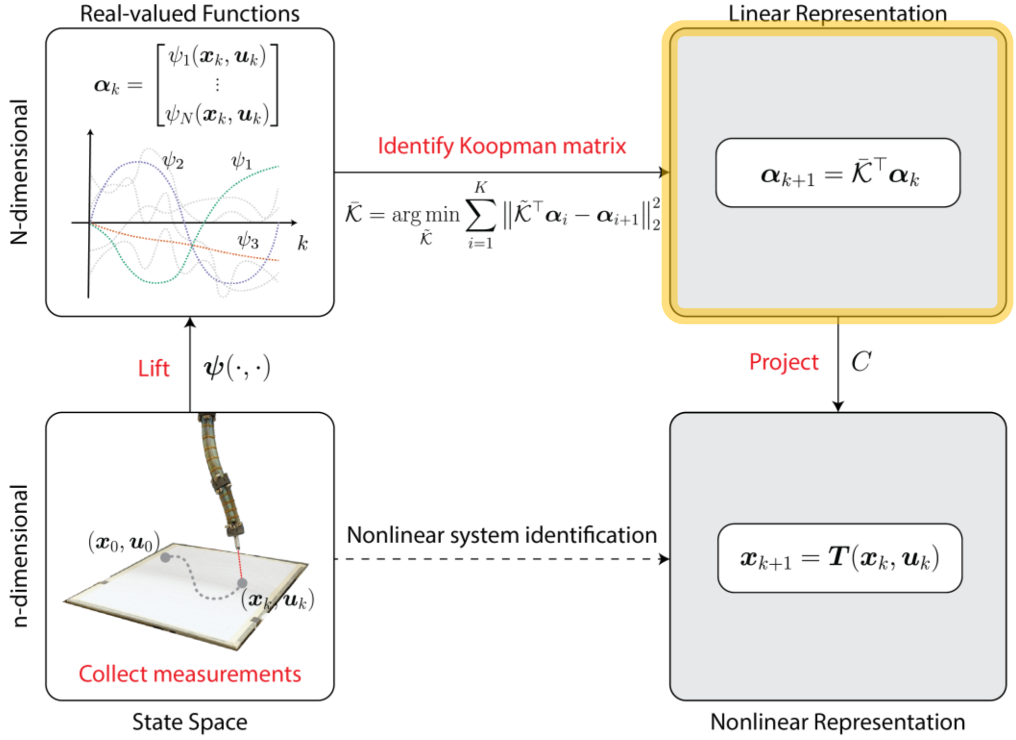
Models are constructed from the Koopman matrix



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Linear Model (decomposition):

$$\bar{\mathcal{K}}^\top = \begin{bmatrix} A & B \\ \vdots & \vdots \end{bmatrix} \longrightarrow \boldsymbol{z}(\boldsymbol{x}_{k+1}) = A\boldsymbol{z}(\boldsymbol{x}_k) + B\boldsymbol{u}_k$$

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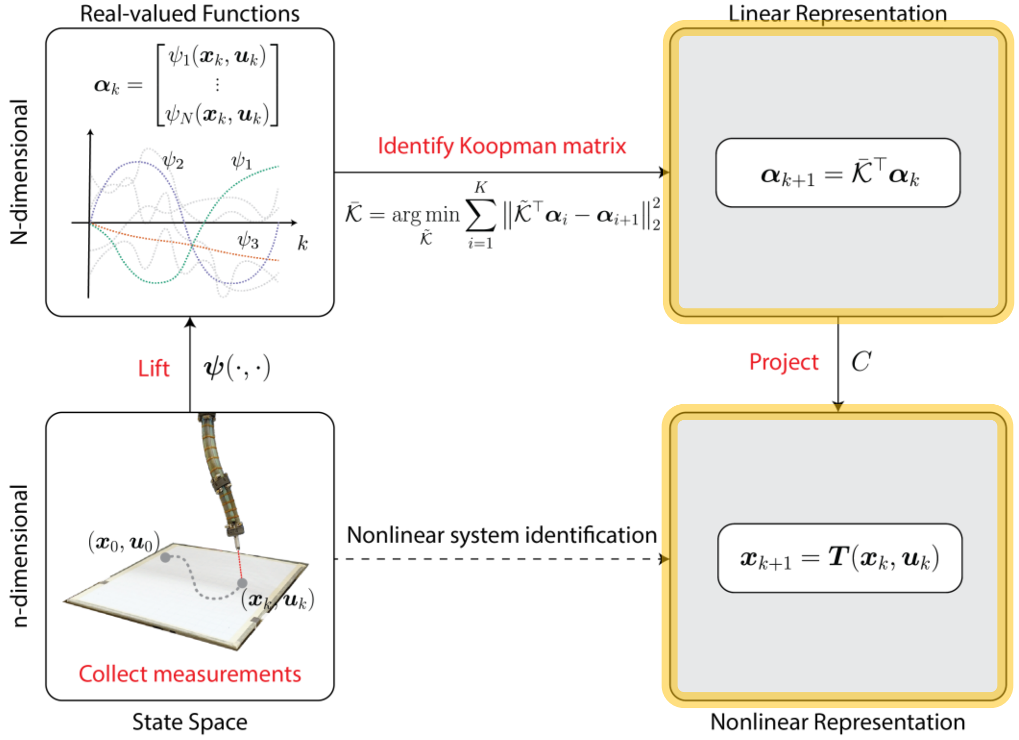
Models are constructed from the Koopman matrix



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Linear Model (decomposition):

$$\bar{\mathcal{K}}^\top = \begin{bmatrix} A & B \\ \vdots & \vdots \end{bmatrix} \longrightarrow \boldsymbol{z}(\boldsymbol{x}_{k+1}) = A\boldsymbol{z}(\boldsymbol{x}_k) + B\boldsymbol{u}_k$$

Nonlinear Model (projection):

$$\boldsymbol{T} = C\bar{\mathcal{K}}^\top \boldsymbol{\psi} \quad \longrightarrow \quad \boldsymbol{x}_{k+1} = \boldsymbol{T}(\boldsymbol{x}_k, \boldsymbol{u}_k)$$

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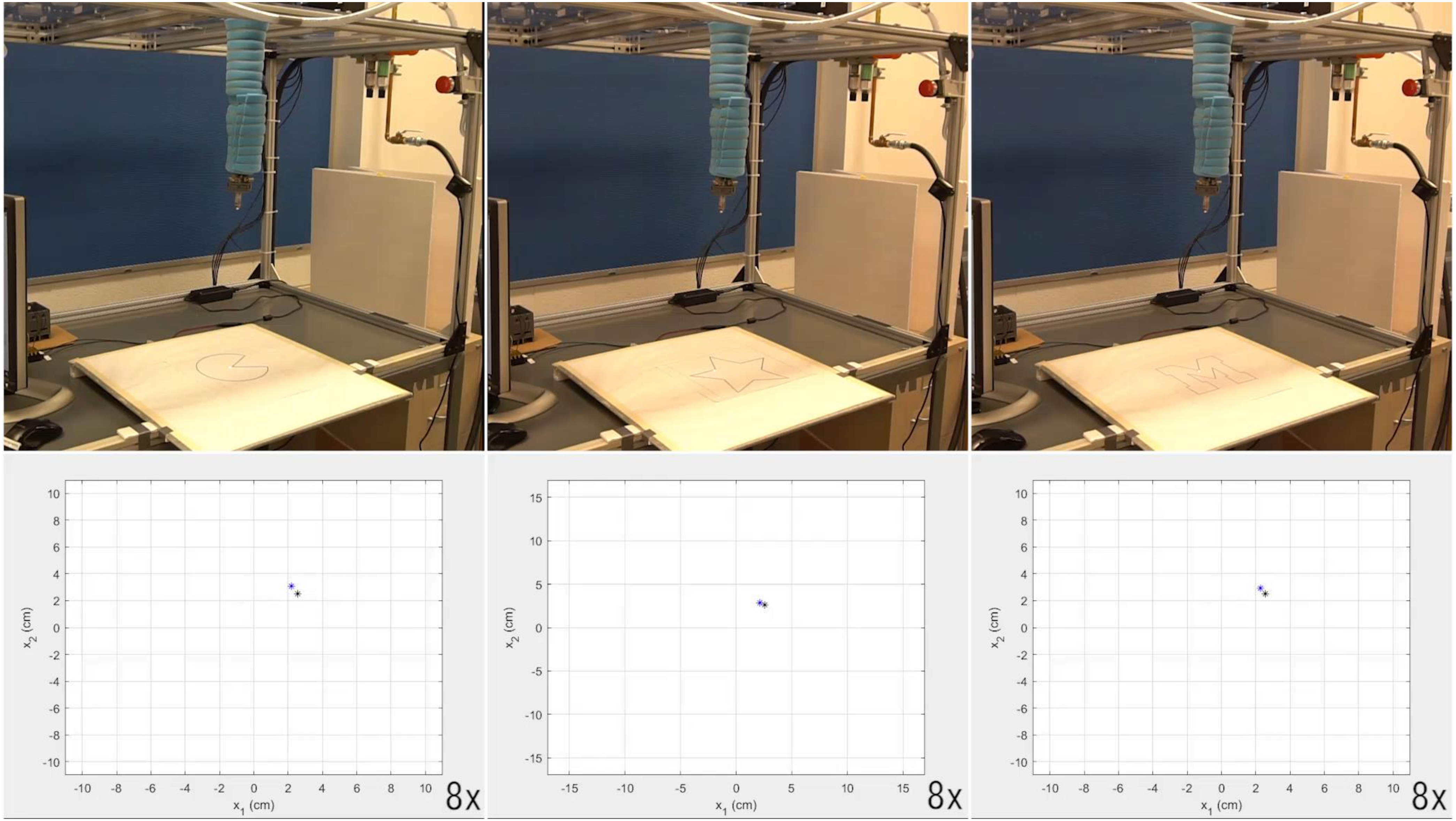
MPC controller uses linear Koopman model to make predictions



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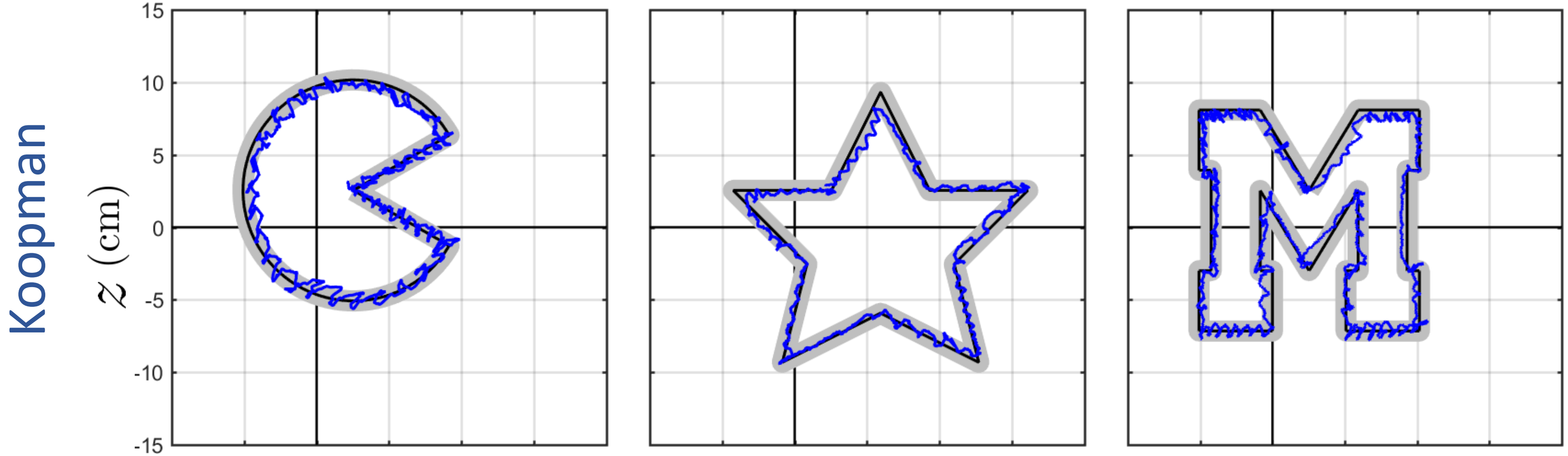
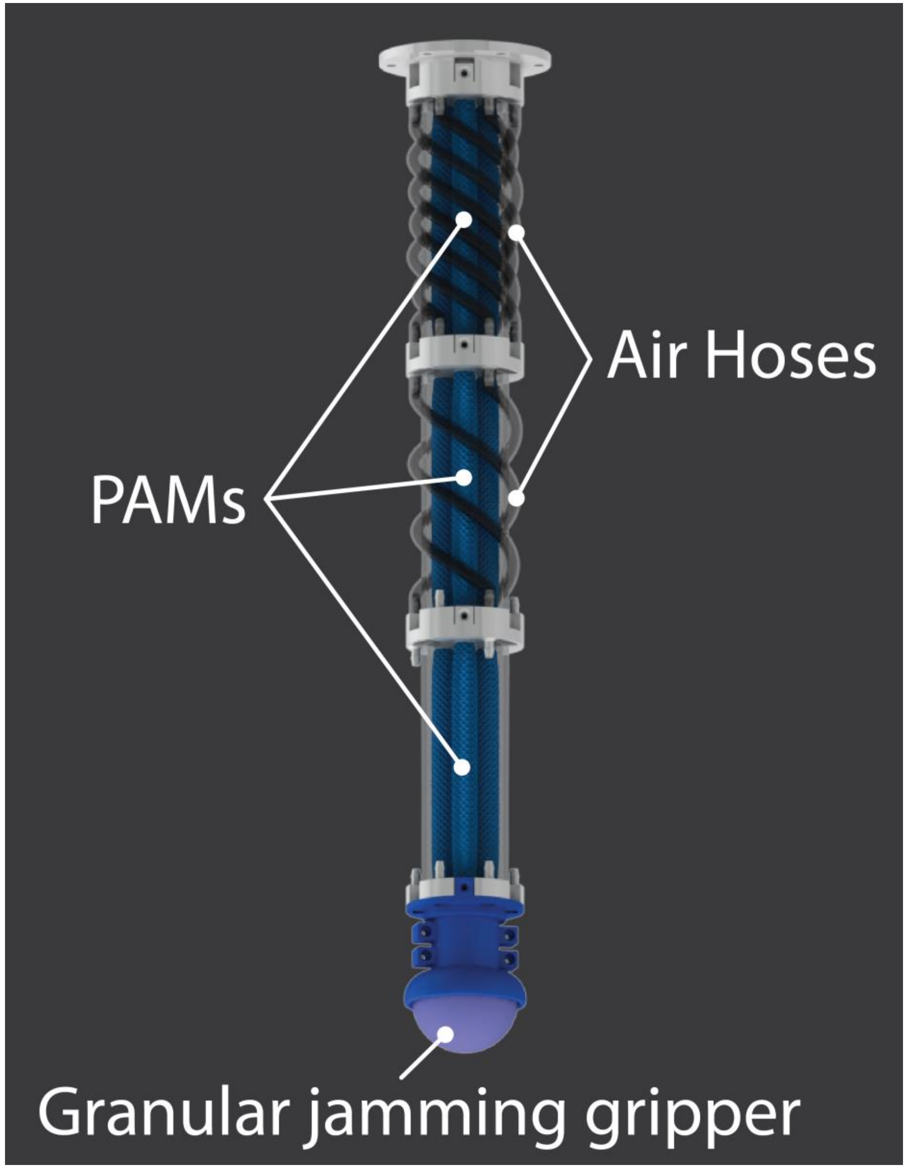
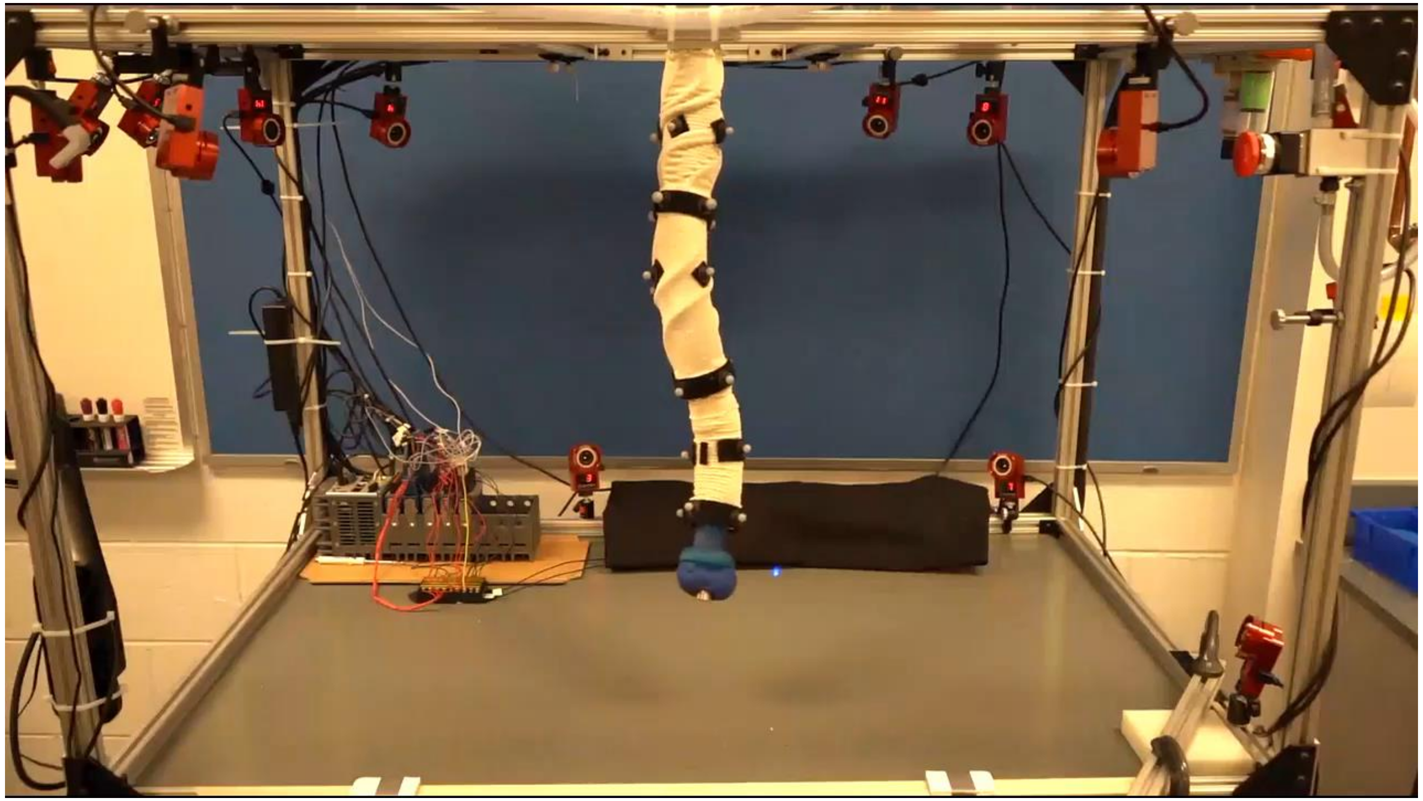
Koopman-based controller outperforms benchmark

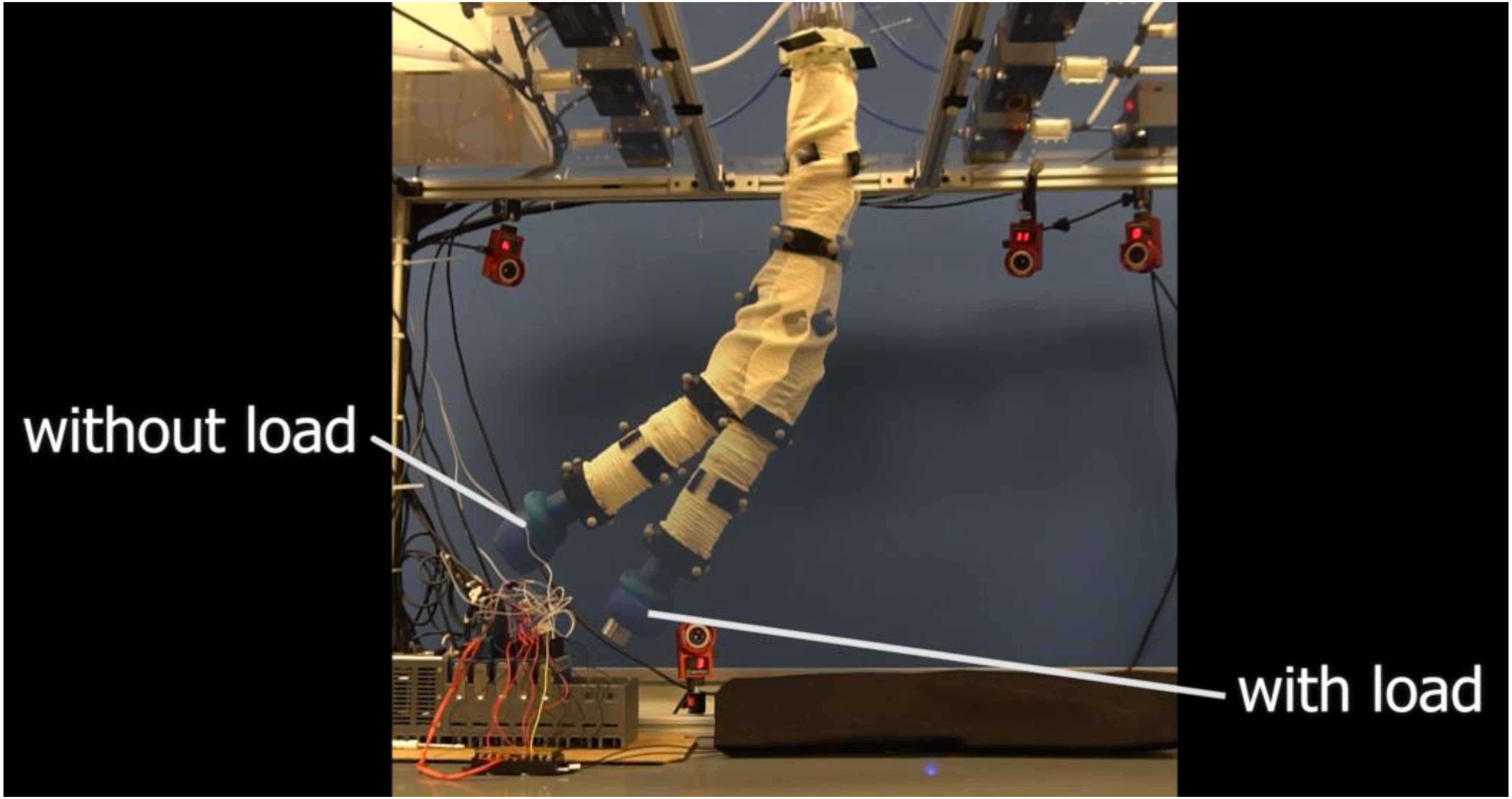


Koopman approach was applied to soft continuum manipulator





Soft manipulators deform a lot due to loading



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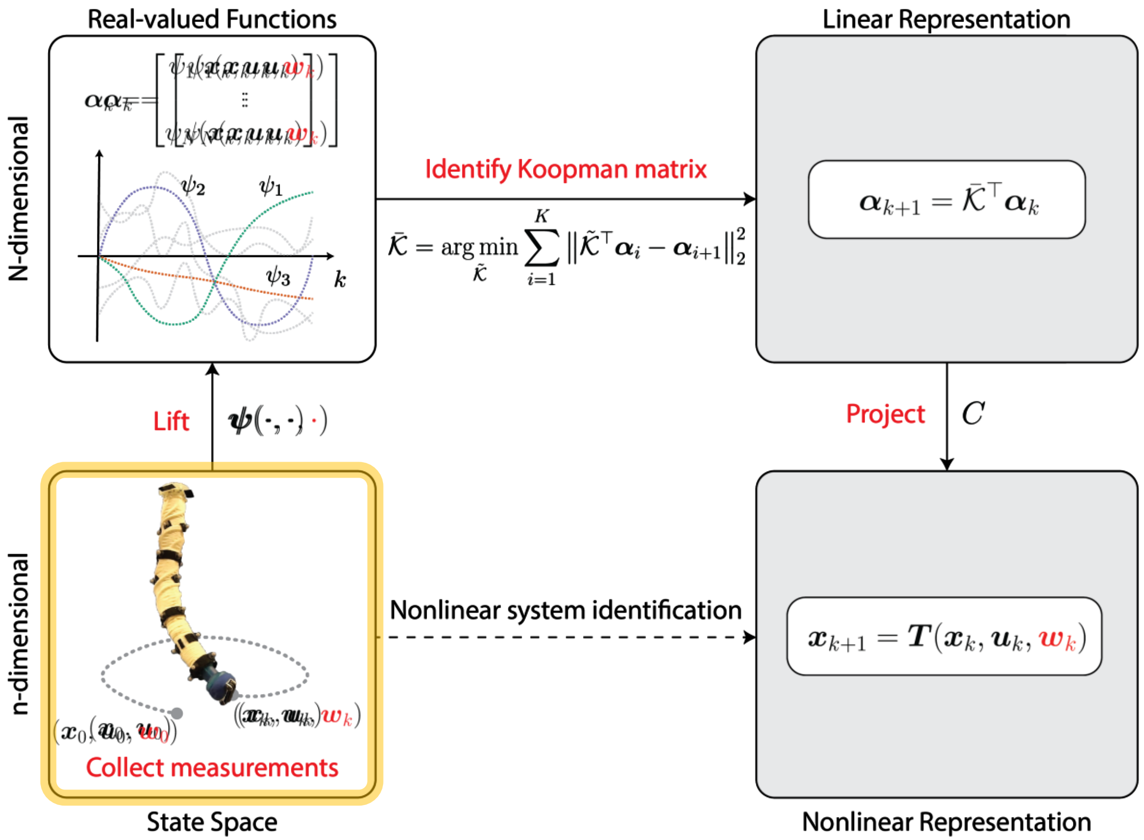
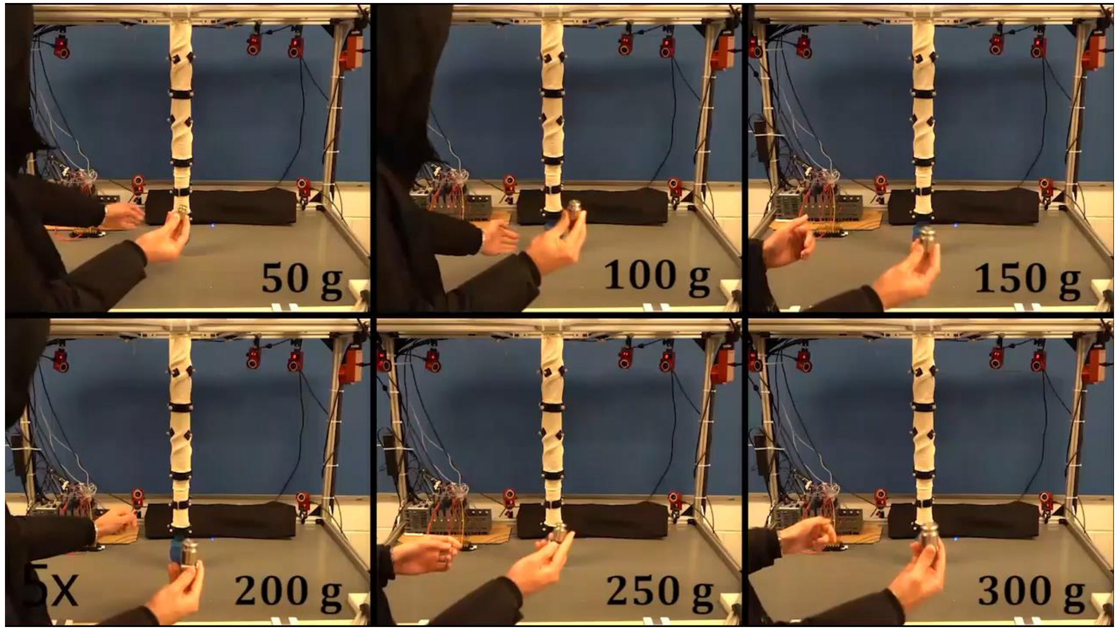
Solution: Incorporate loading into lifted model





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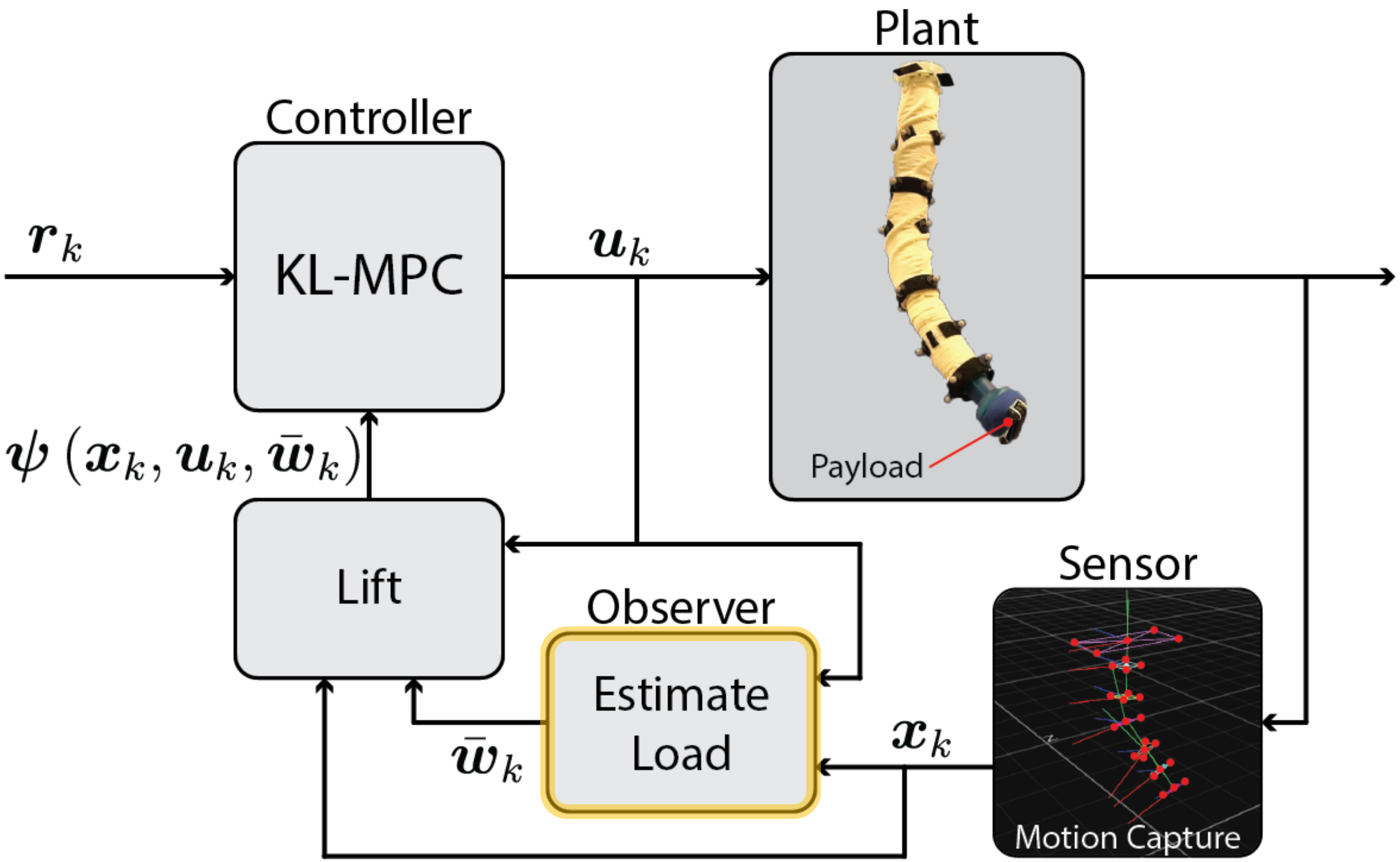
Observer estimates load based on past measurements



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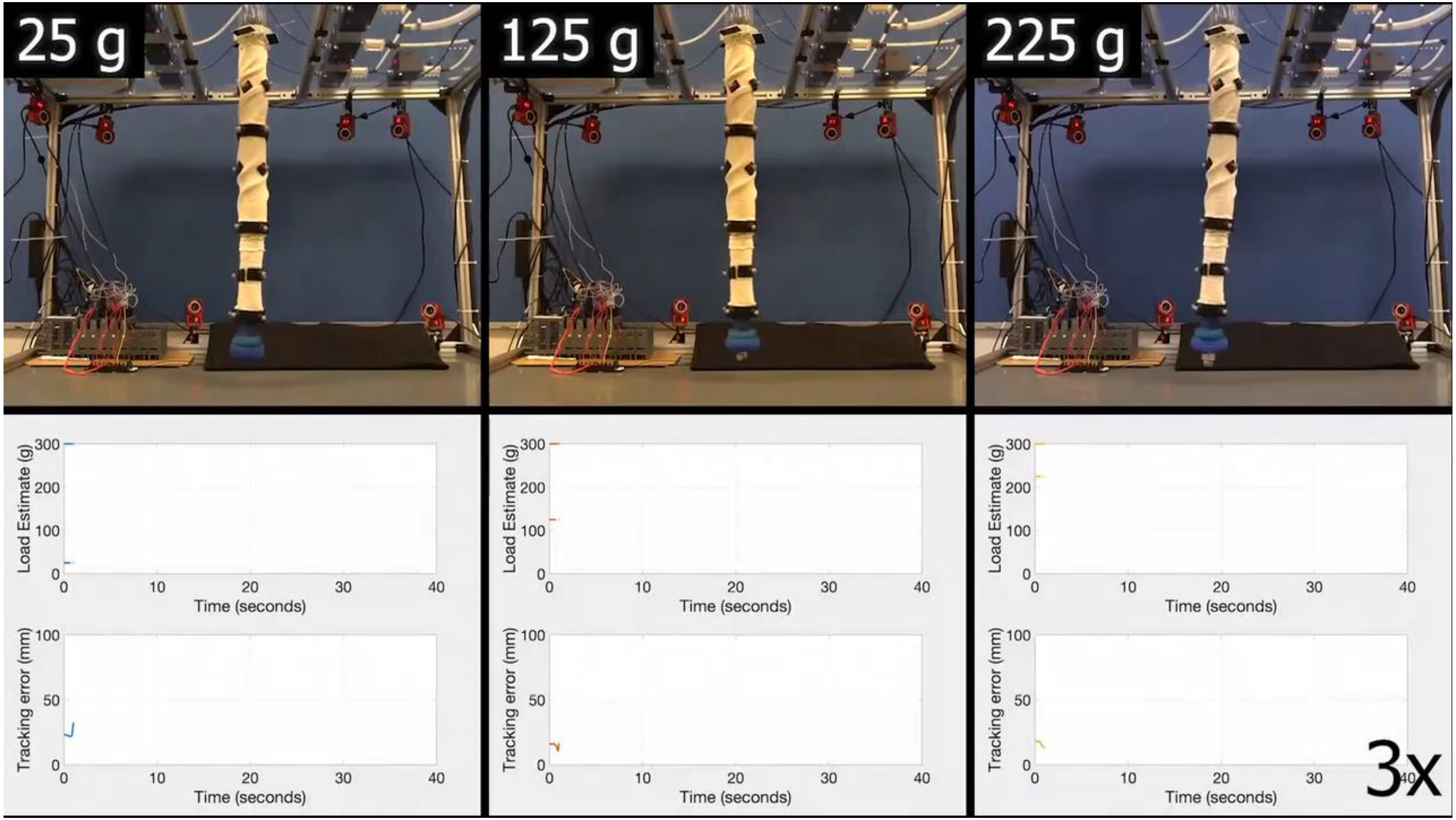
As load estimate error decreases, so does tracking error



Demo: Automated object sorting by mass

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Capabilities and limitations of Koopman modeling framework

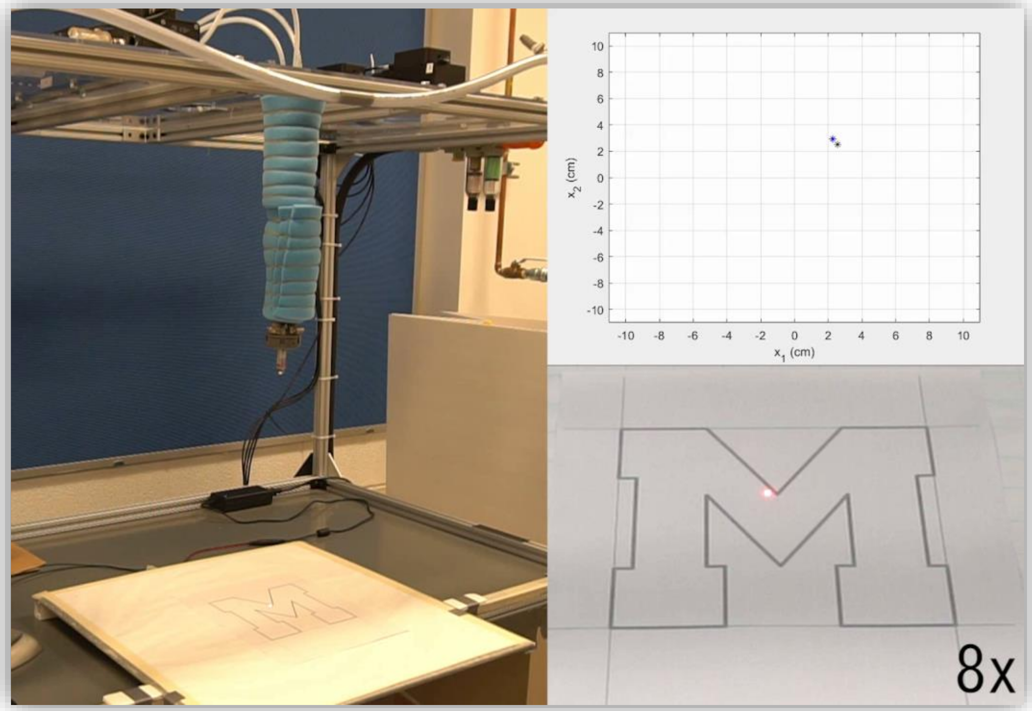
Capabilities

- Enables real-time (linear) control
- Applies to arbitrary soft robots
- Accounts for loading

Limitations (for now)

- “Curse of dimensionality”
- Depends on training data
- Does not account for contact

Thank you!



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