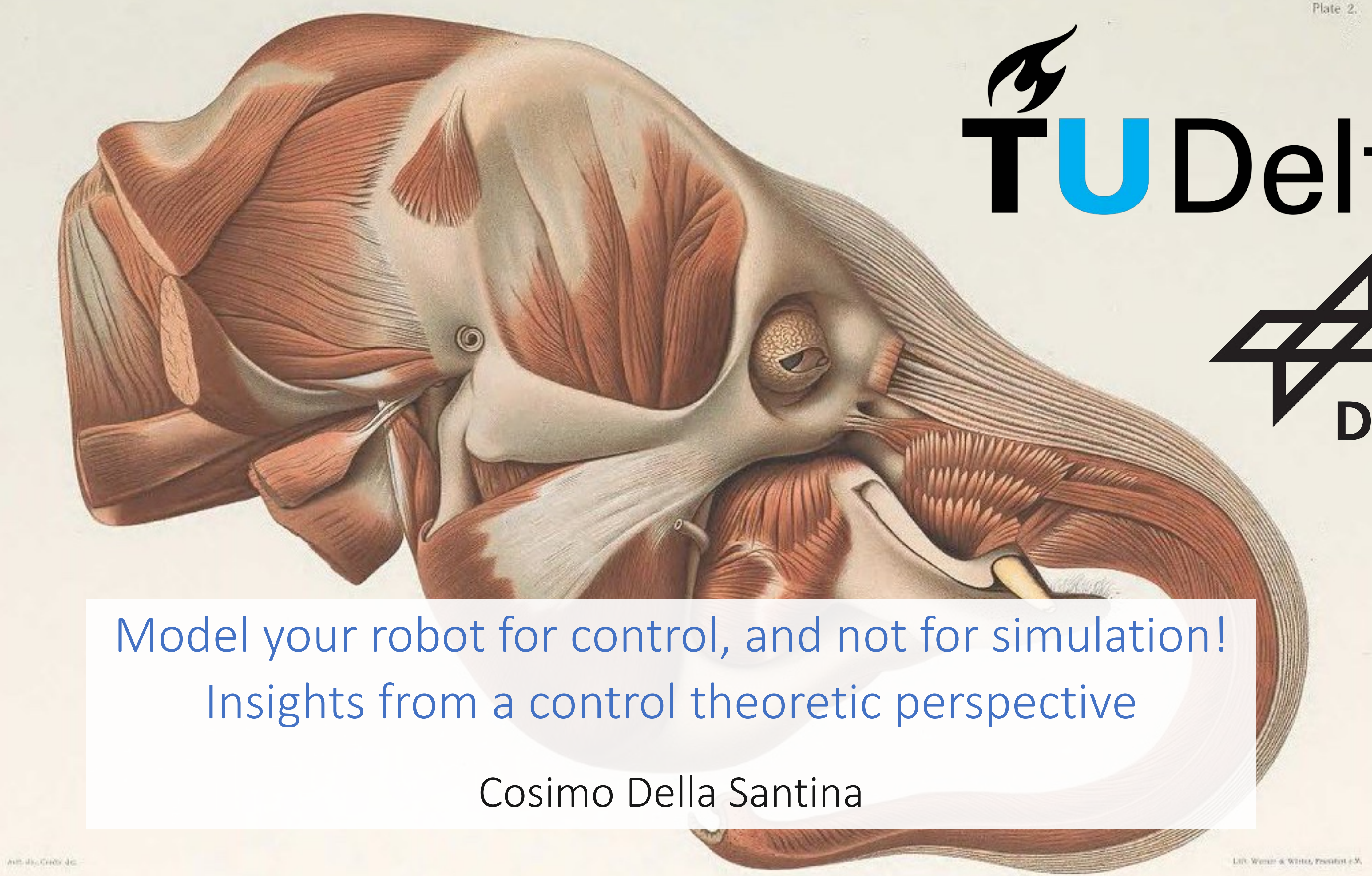
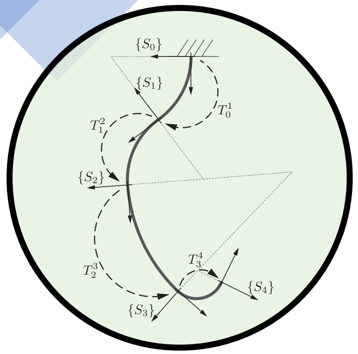
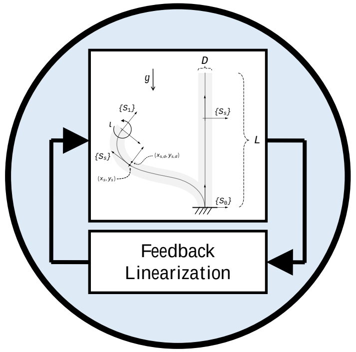


Model your robot for control, and not for simulation!
Insights from a control theoretic perspective

Cosimo Della Santina

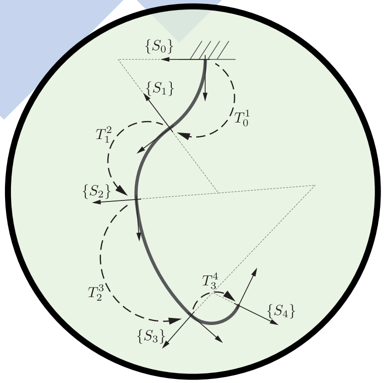


Feedback Model Based Control
Is Robust to Rough Approximations



If You Want to Dig More
Do That in a Control Oriented Way

If You Want to Stick to the Simple Model,
Consired Control-Driven Ways to Improve It



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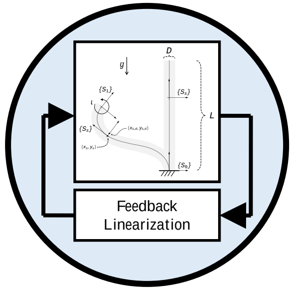

If You Want to Dig More
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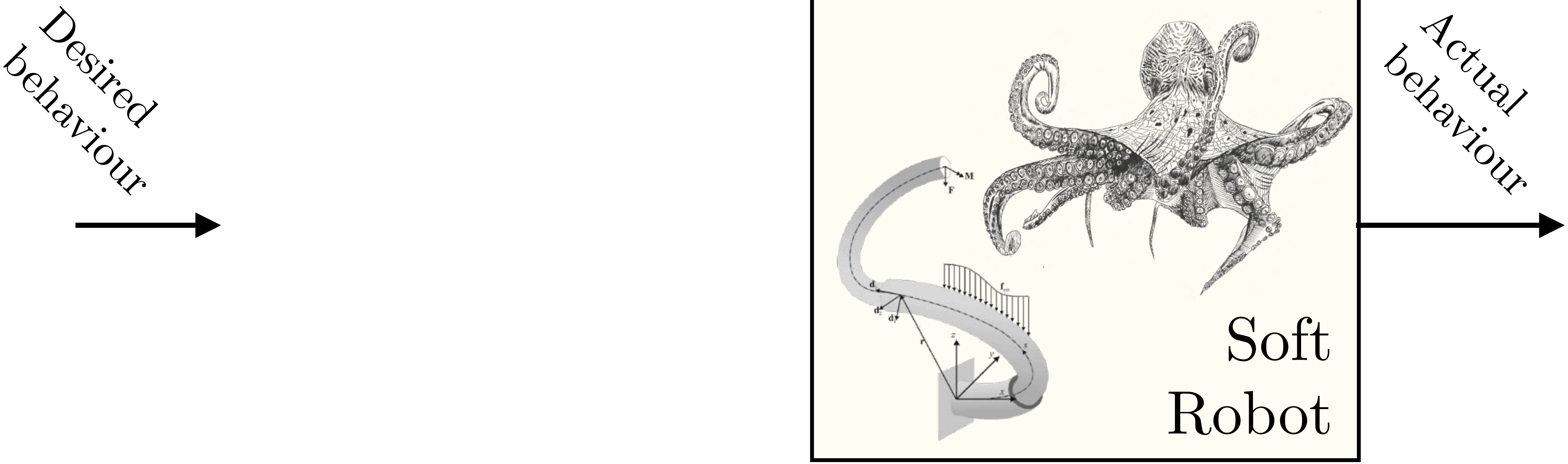
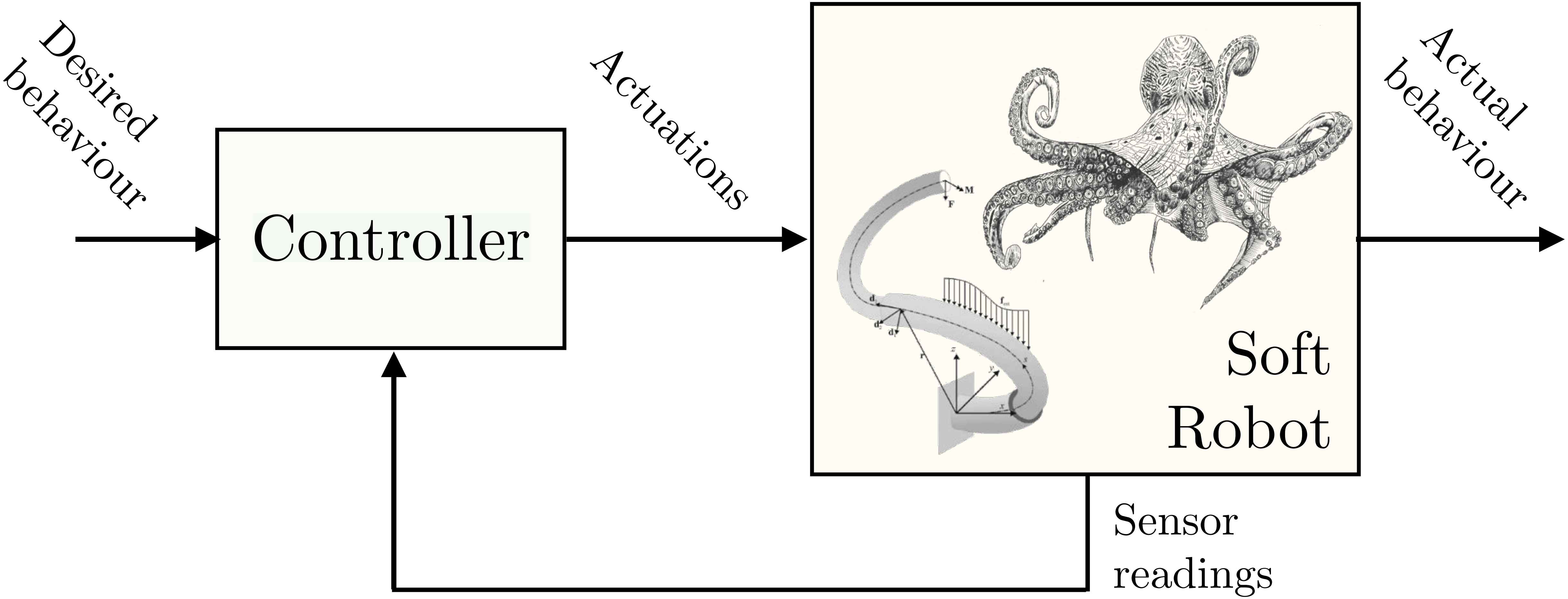
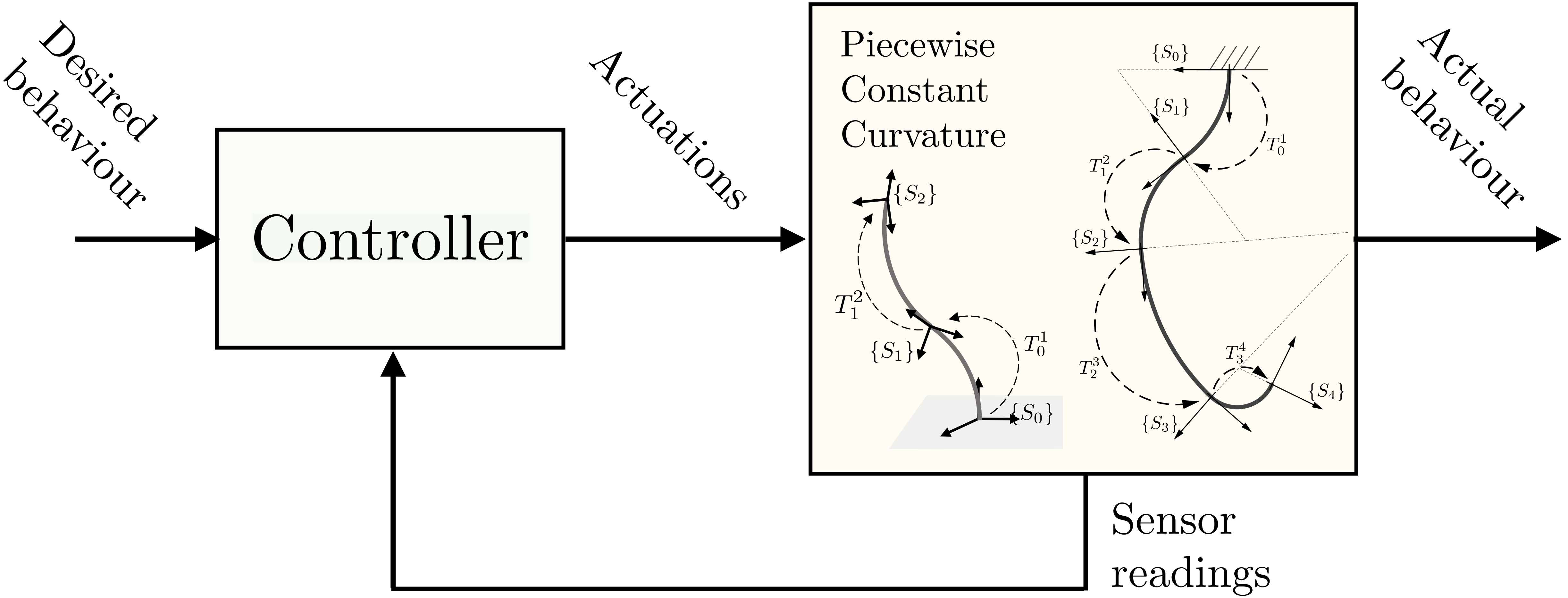
A Grand Challenge Within Soft Robotics



A Grand Challenge Within Soft Robotics

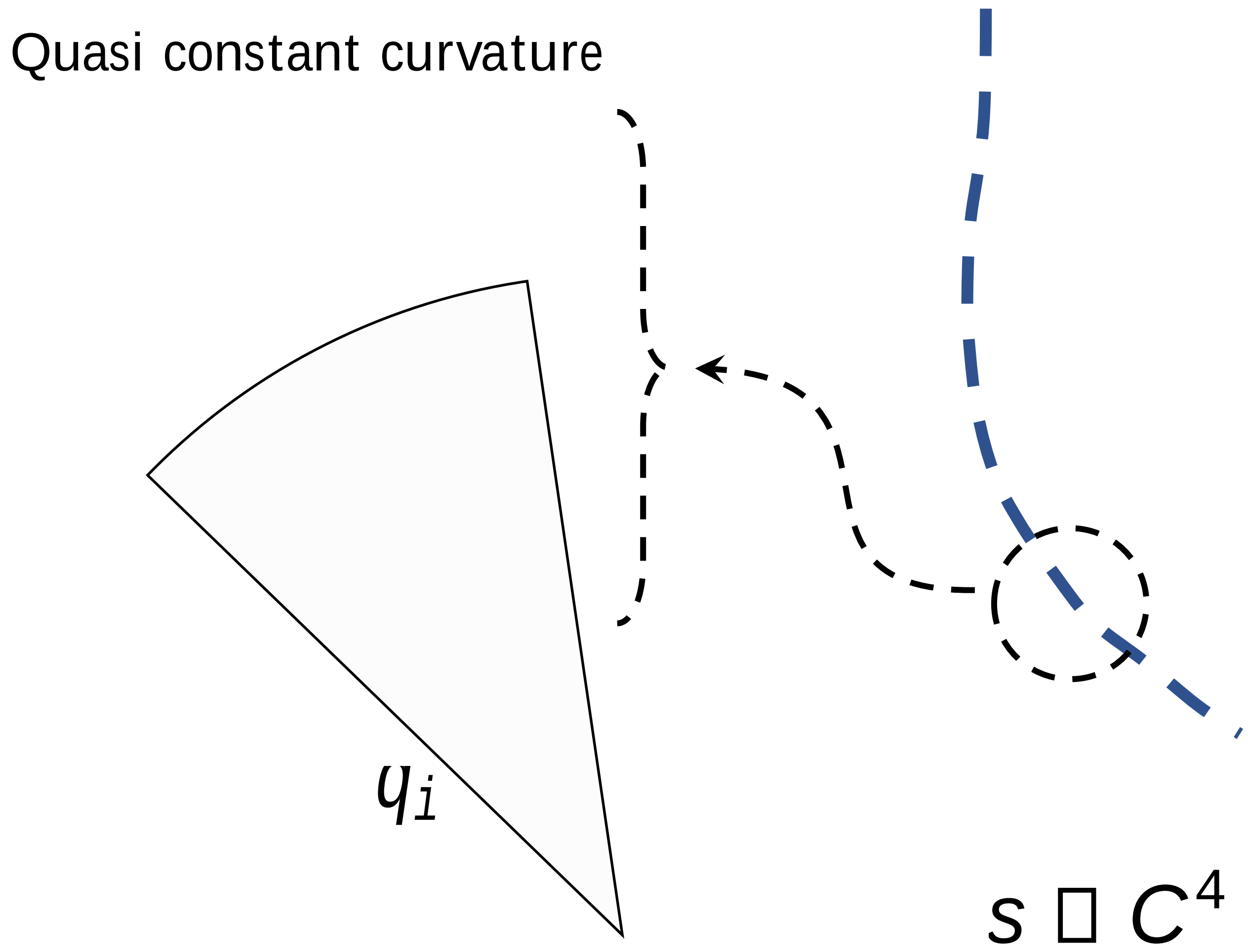


A Grand Challenge Within Soft Robotics



Wang et al. "Visual Servoing of Soft Robot Manipulator in Constrained Environments With an Adaptive Controller" TOM 2017
Falkenhahn et al. "Model-based feedforward position control of constant curvature continuum robots using feedback linearization" ICRA 2015

Torques >> Pure Forces



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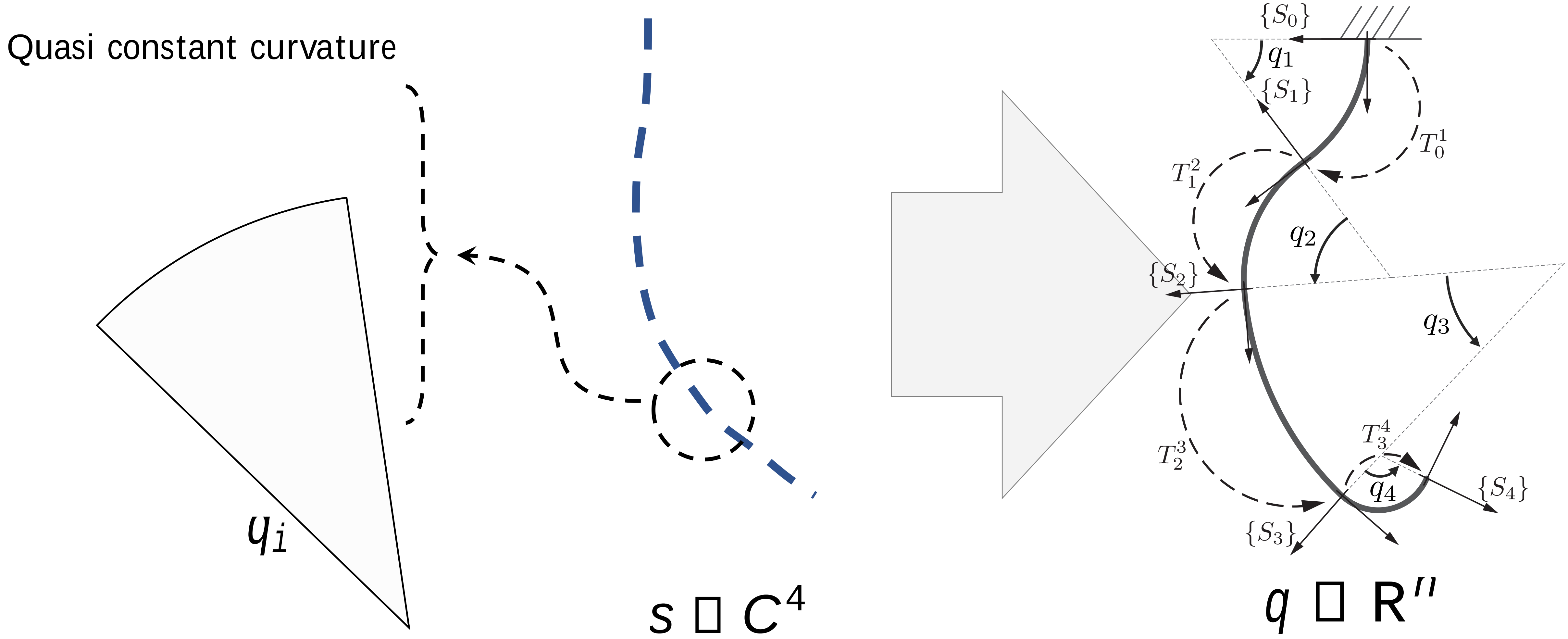

Wang et al. "Visual Servoing of Soft Robot Manipulator in Constrained Environments With an Adaptive Controller" TOM 2017
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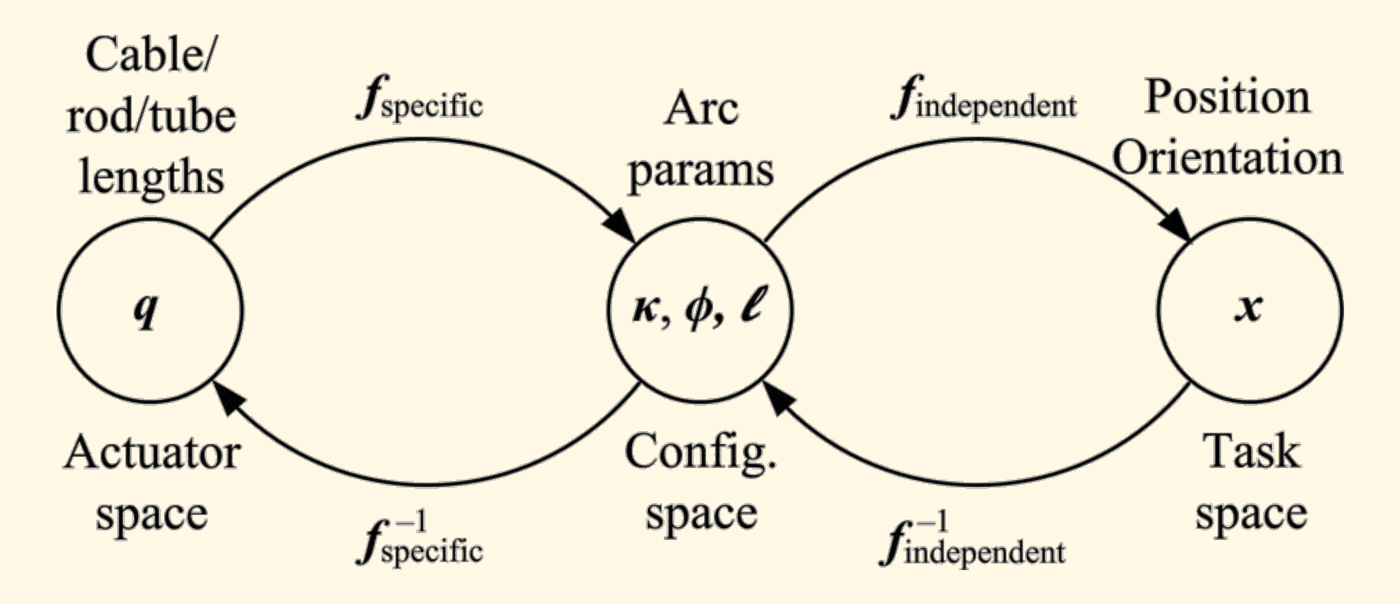
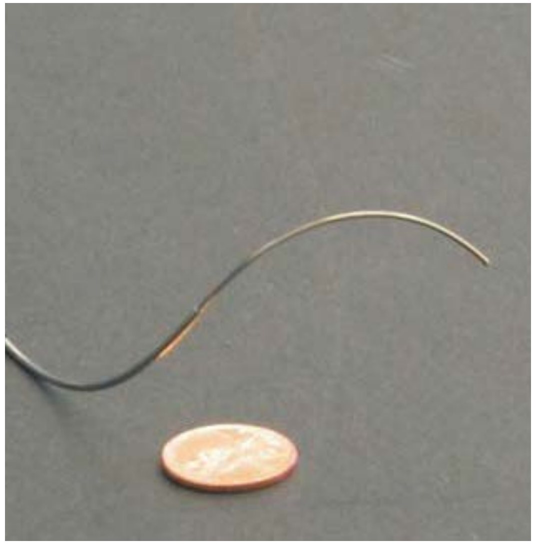
Webster and Jones “Design and Kinematic Modelingof Constant Curvature ContinuumRobots: A Review” IJRR

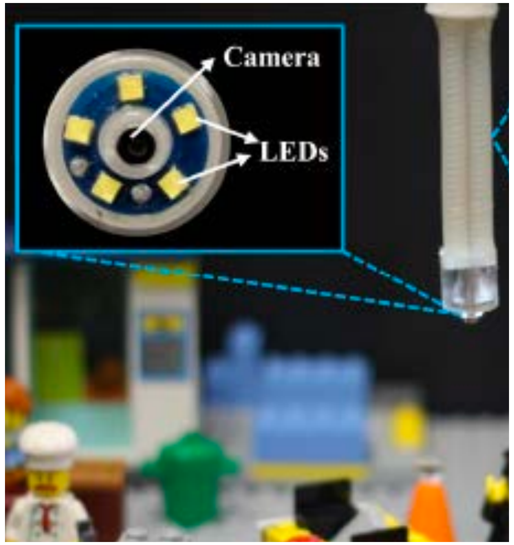


Quasi-static kinematic control



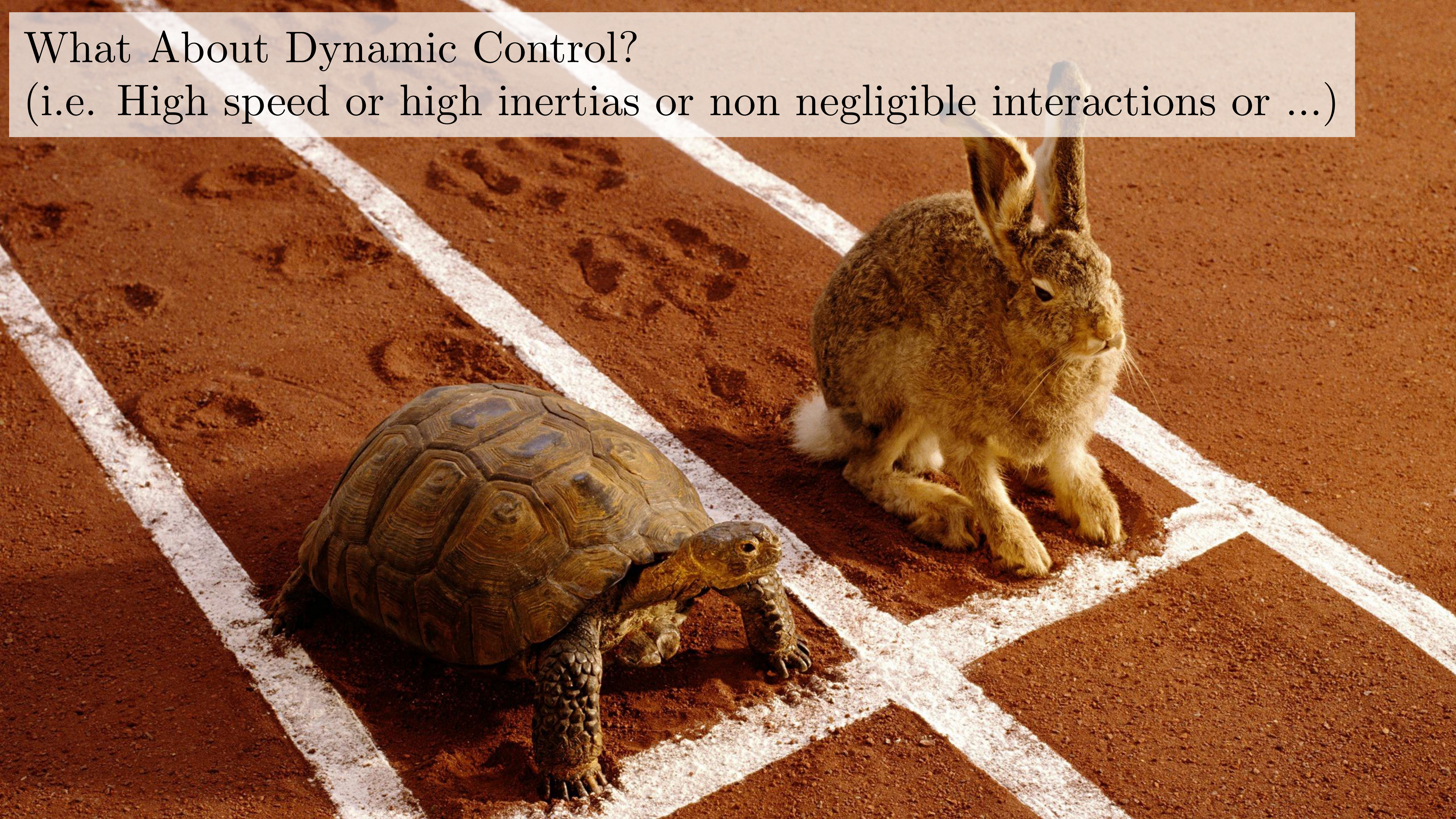
Webster, R. J., III, Romano, J. M. and Cowan, N. J. (2009). Mechanics of precurved-tube continuum robots. IEEE Transactions on Robotics

Gravagne, I. A., Rahn, C. and Walker, I. D. (2003). Largedeflection dynamics and control for planar continuum robots. IEEE/ASME Transactions on Mechatronics, 8(2): 299–307.

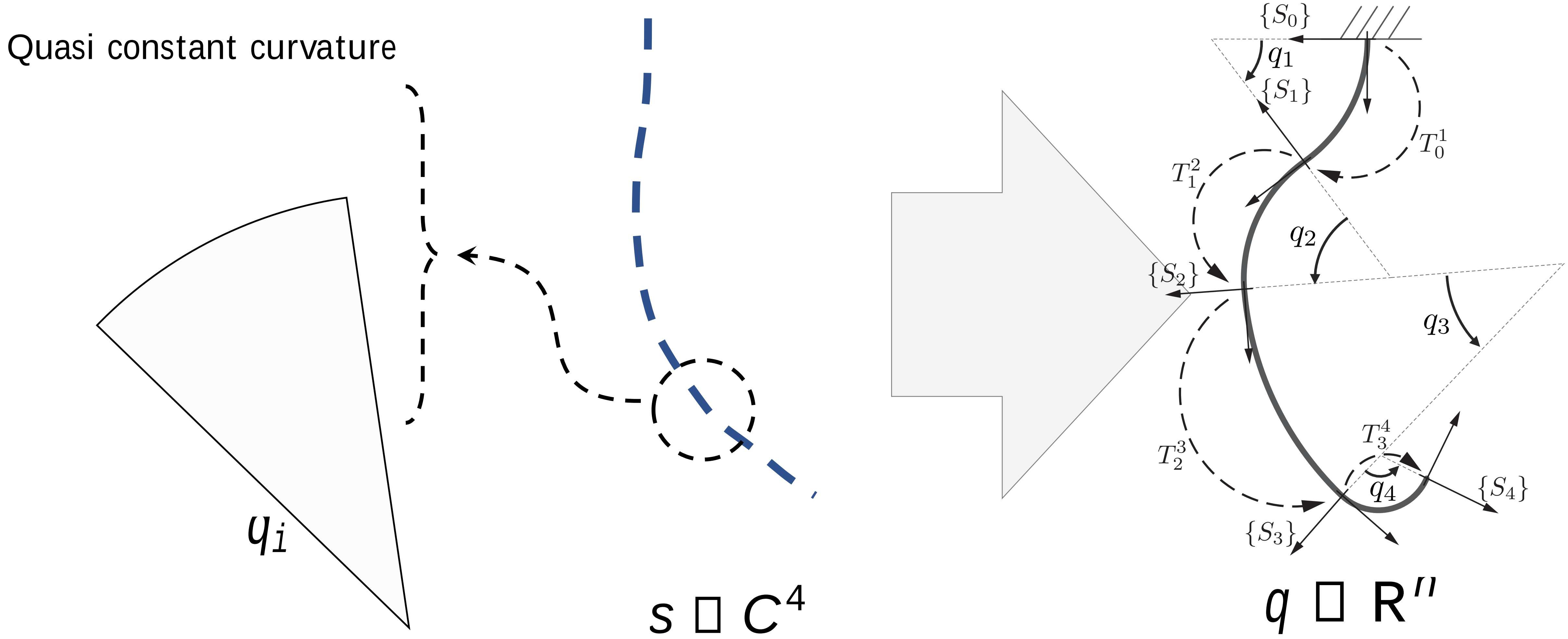


Fang, Ge, et al. "Vision-based online learning kinematic control for soft robots using local gaussian process regression." *IEEE Robotics and Automation Letters* 4.2 (2019): 1194-1201.

What About Dynamic Control?
(i.e. High speed or high inertias or non negligible interactions or ...)



Torques >> Pure Forces



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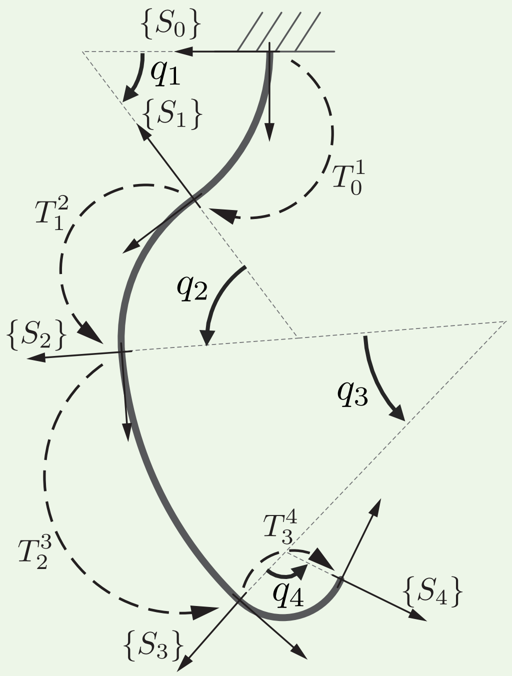
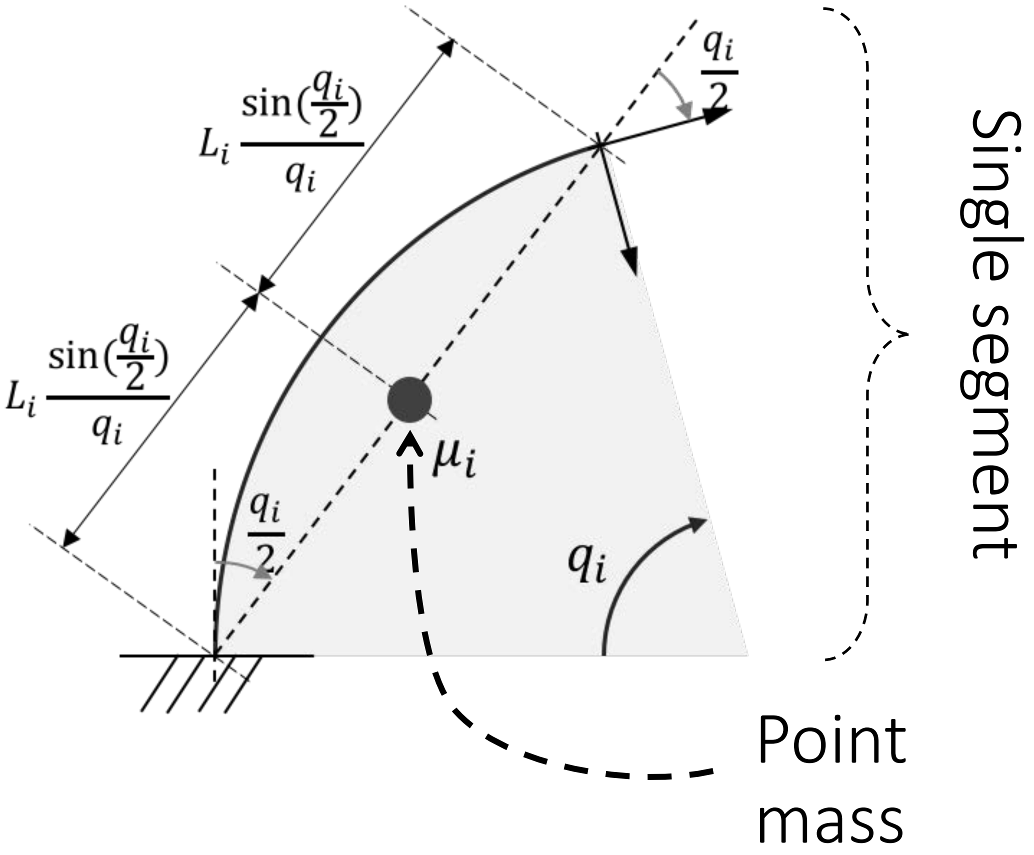
Hypothesis 1 (kinematics):
SR's can be approximated as a series of n segments with constant curvature (CC)



Hypothesis 2 (inertia):
The inertia of each segment is described by an equivalent point mass.



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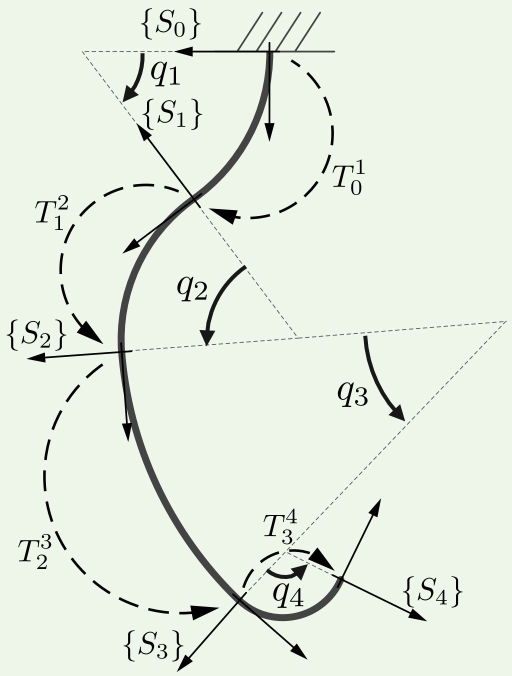

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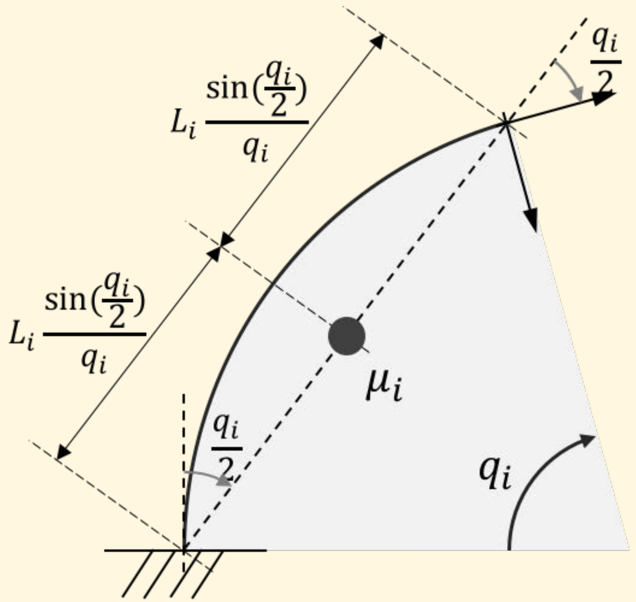


Hypothesis 3 (impedance): continuous distribution of infinitesimal springs and dampers

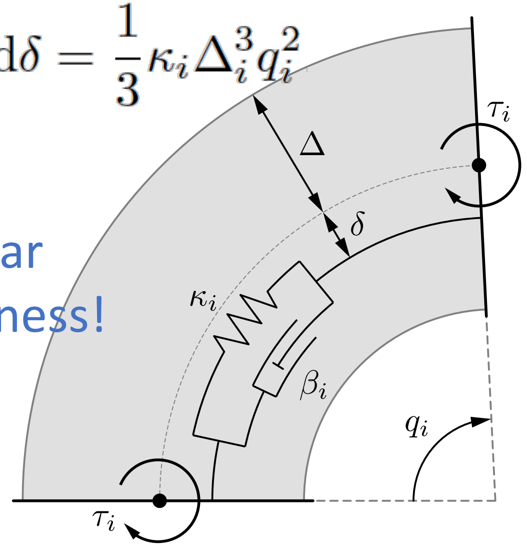
$$L_{\delta,i}(q_i) = (\rho_i - \delta)\, q_i = (\frac{L_i}{q_i} - \delta)\, q_i$$

$$E_{\delta,i}(q_i) = \frac{1}{2}\kappa_i (L_{\delta,i}(0) - L_{\delta,i}(q_i))^2 = \frac{1}{2}\kappa_i \delta^2 q_i^2 \,.$$

$$E_i(q_i) = \int_{-\Delta_i}^{+\Delta_i} E_{\delta,i}(q_i)\mathrm{d}\delta = \frac{1}{3}\kappa_i \Delta_i^3 q_i^2$$

$$\frac{\partial E_i(q_i)}{\partial q_i} = \frac{2}{3}\kappa_i \Delta_i^3 q_i$$

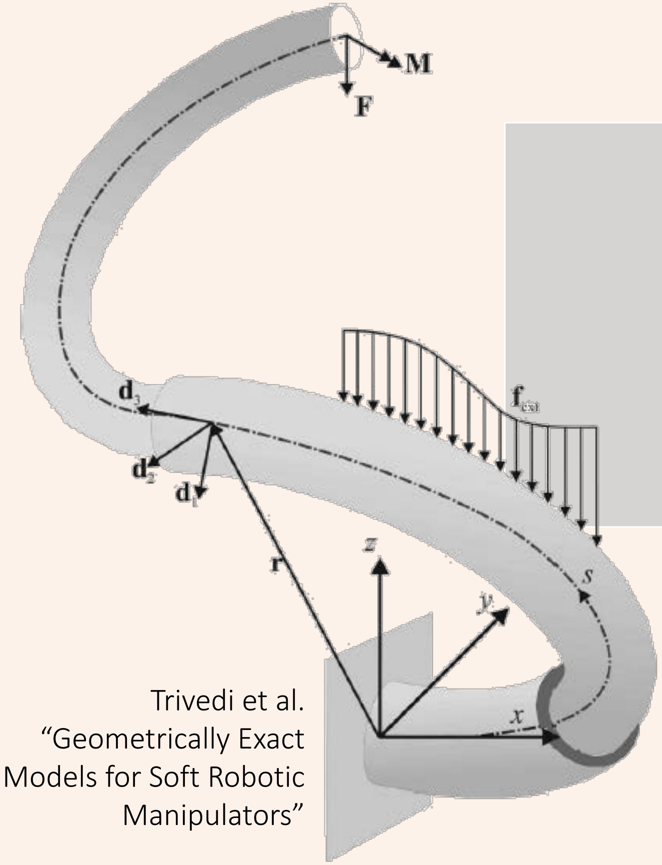
Linear stiffness!



Continuum world

$$(\mathbf{n} - F_{\text{ex}}\mathbf{d}_3)_{,s} + \mathbf{f} = \rho A_t \mathbf{r}_{,tt}$$

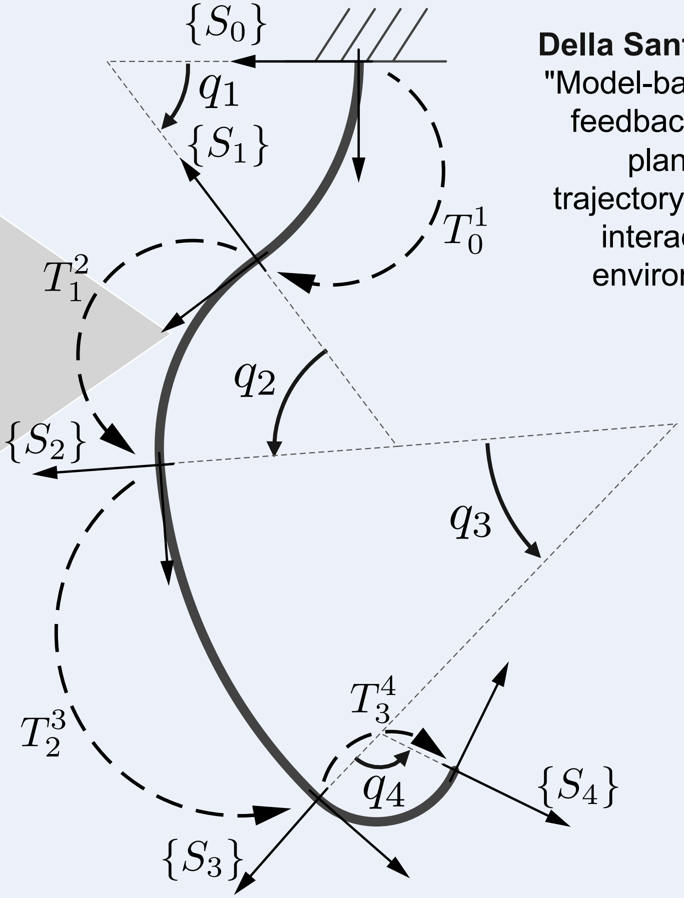
$$\mathbf{m}_{,s} + \mathbf{r}_{,s} \times \mathbf{n} = \mathbf{J}\omega_{,t}$$



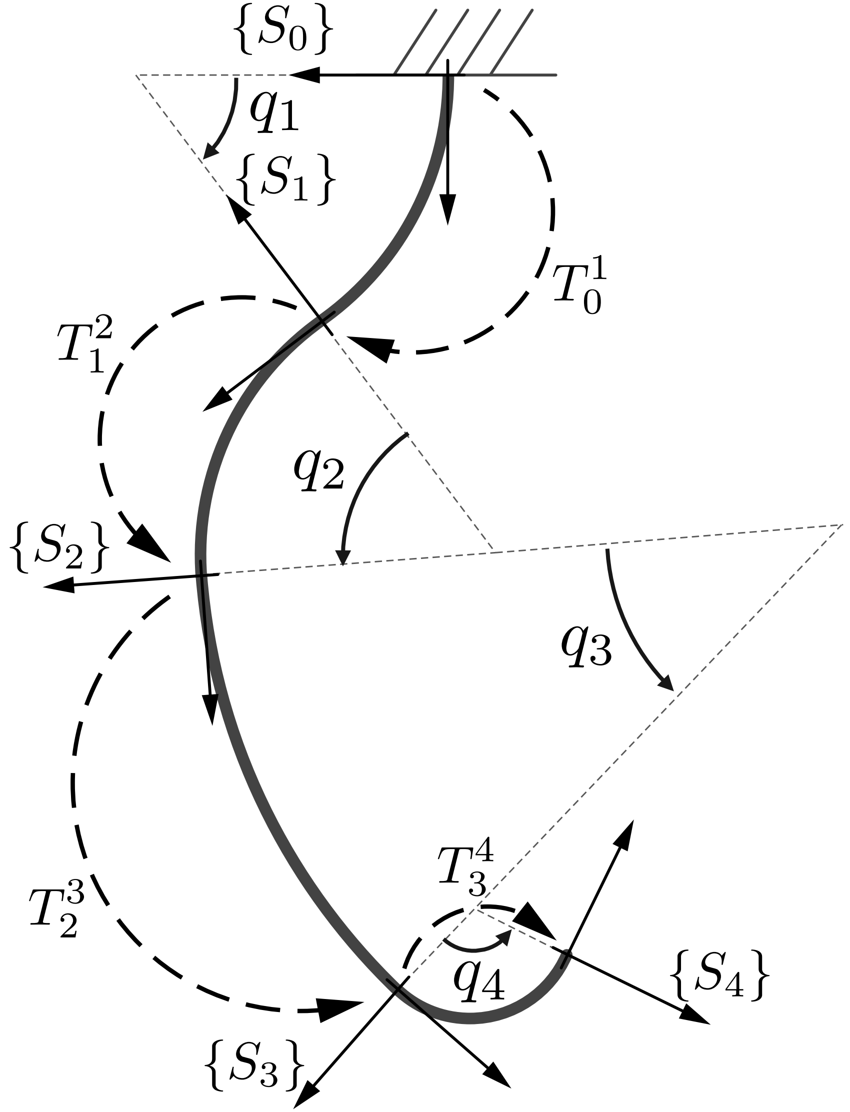
Trivedi et al. "Geometrically Exact Models for Soft Robotic Manipulators"

Discrete world

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) +$$
$$Kq + D\dot{q} = \tau$$



Della Santina C., et al. "Model-based dynamic feedback control of a planar soft robot: trajectory tracking and interaction with the environment." IJRR

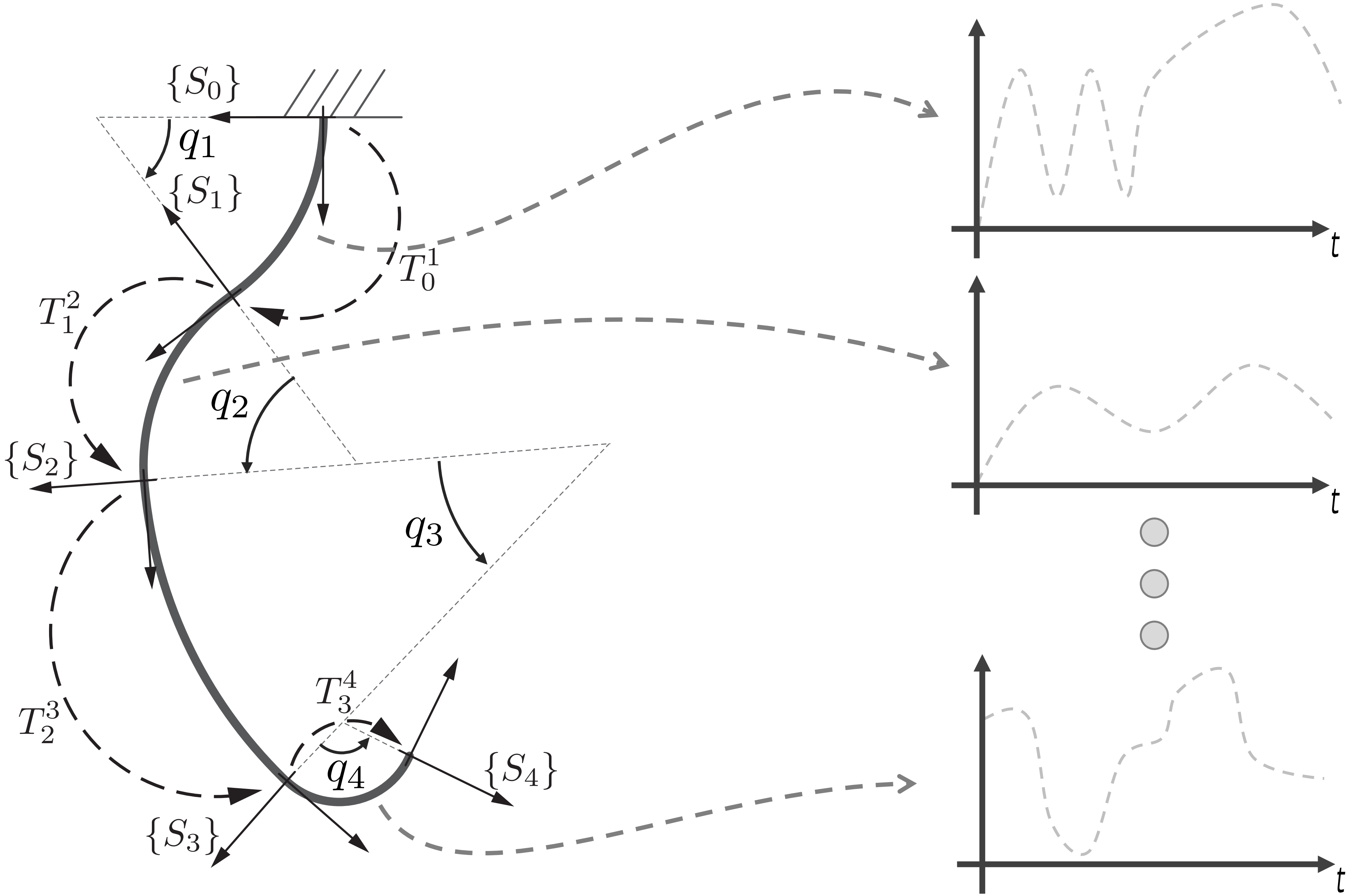


Della Santina C., et al. "Model-based dynamic feedback control of a planar soft robot: trajectory tracking and interaction with the environment." IJRR

Trajectory tracking in curvature space

$\dot{\bar{q}} = 0$

$\ddot{\bar{q}} = 0$



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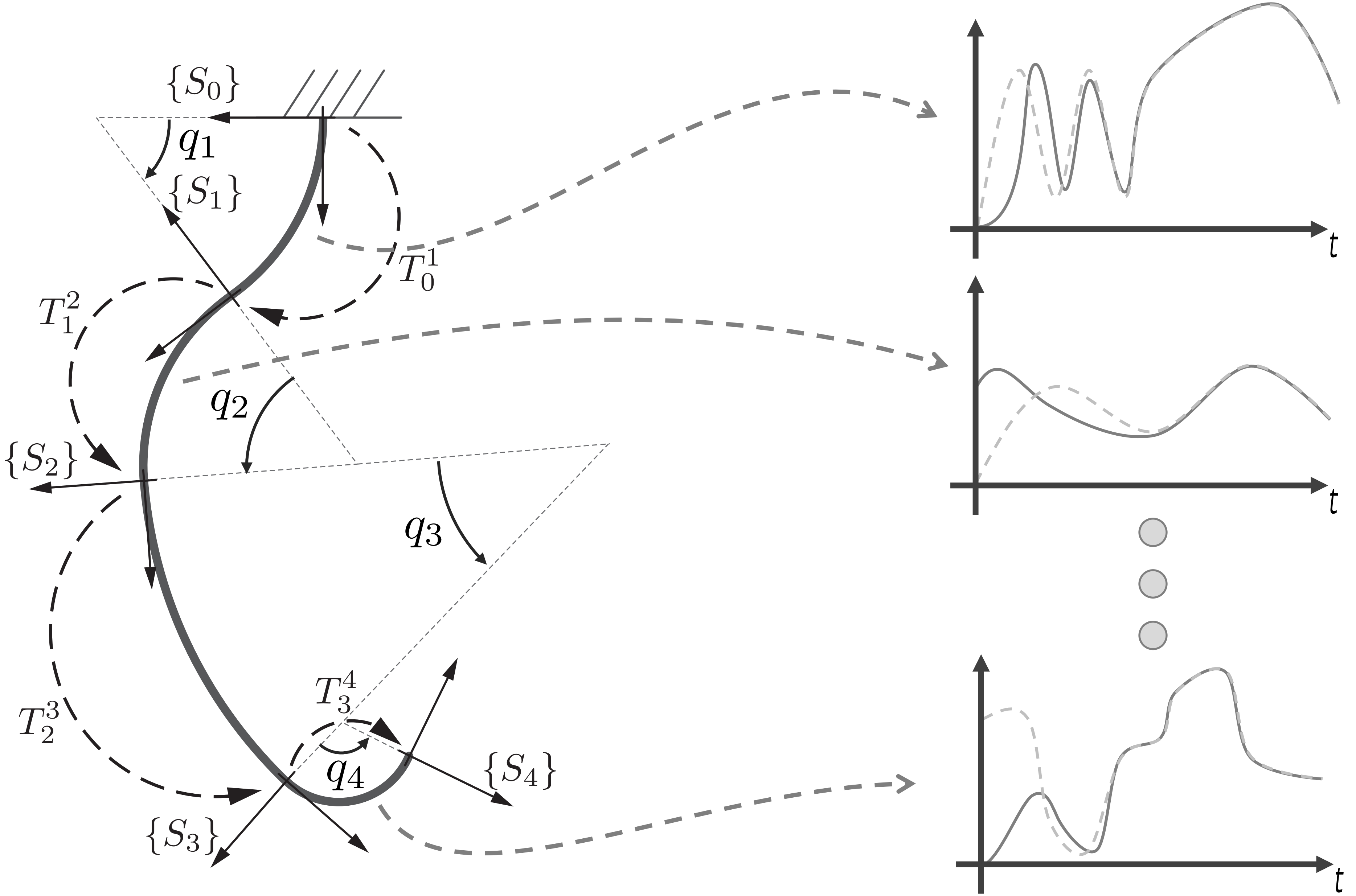

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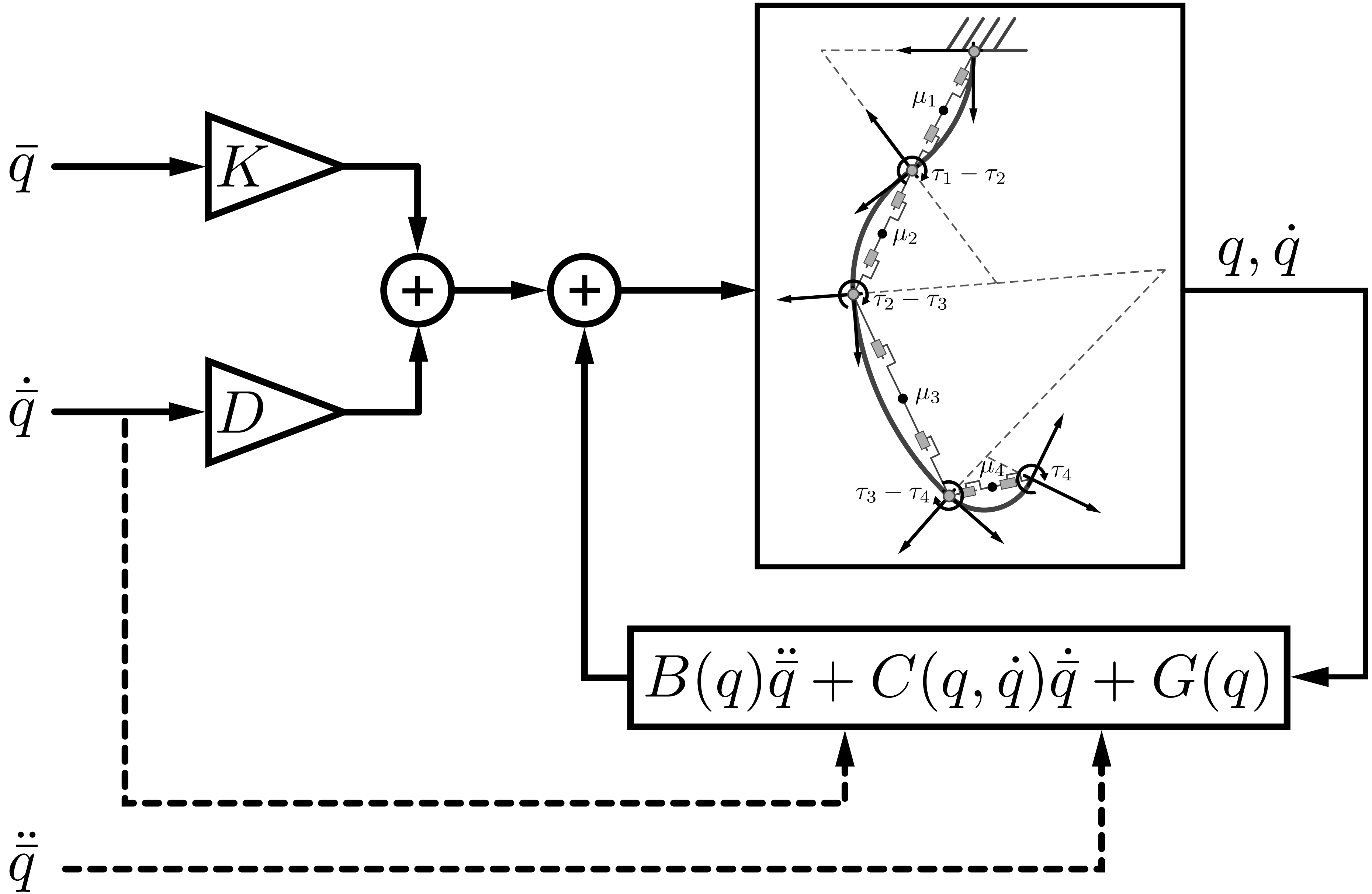
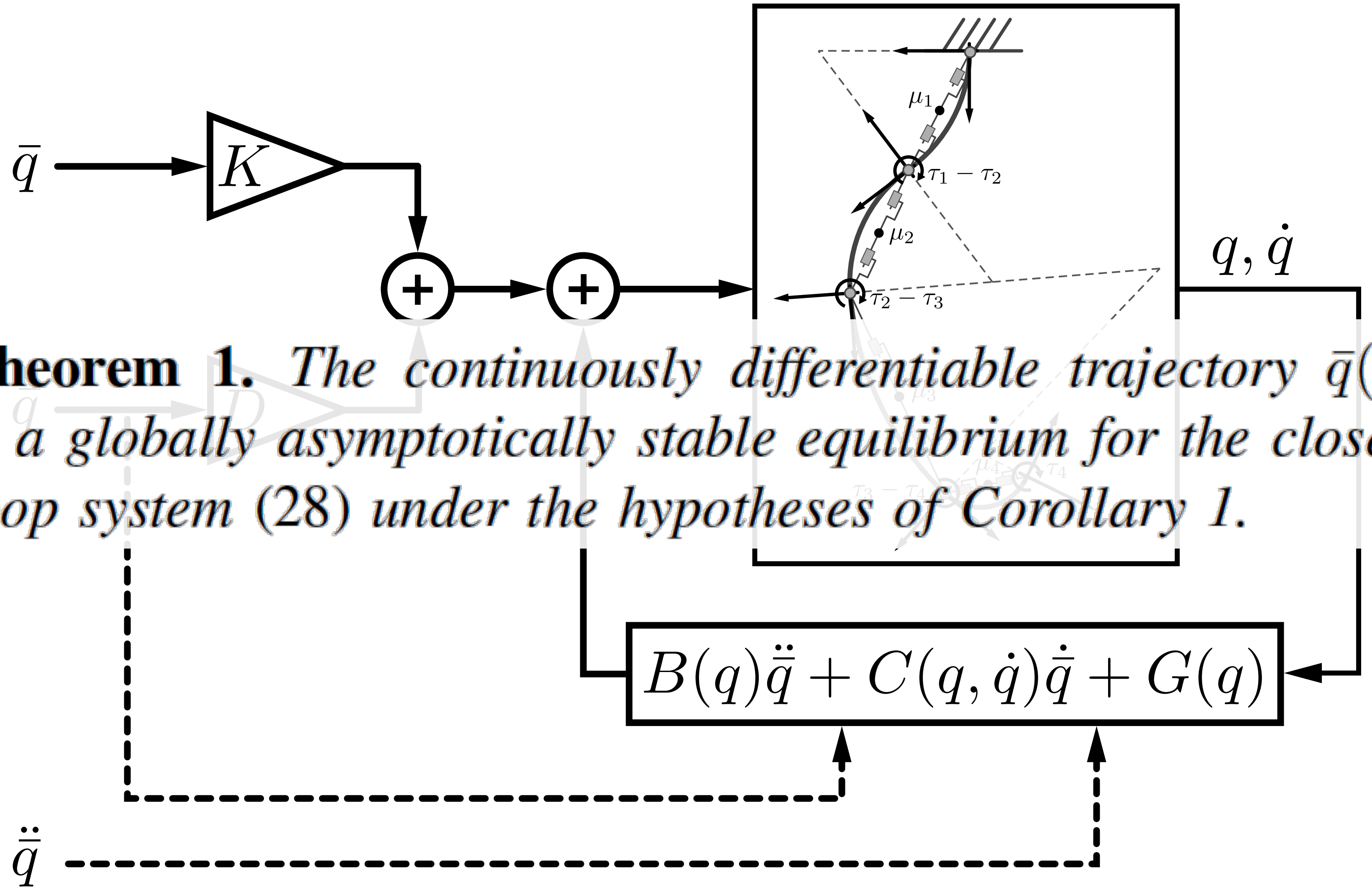
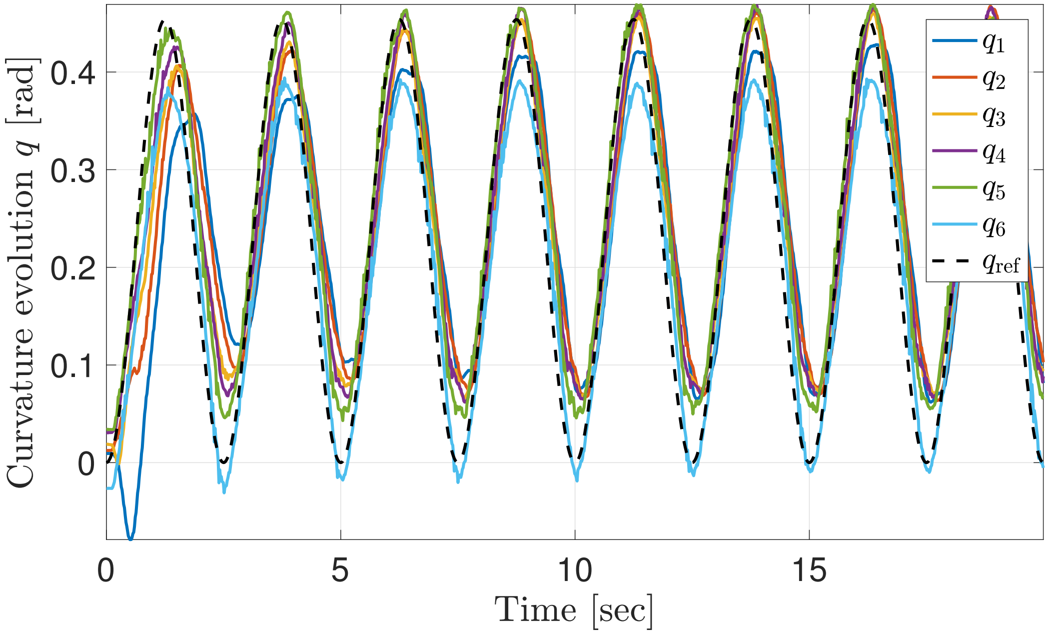


$\bar{q}$
K
$\dot{\bar{q}}$
D
$\ddot{\bar{q}}$
$q, \dot{q}$
μ_1
μ_2
μ_3
μ_4
$\tau_1 - \tau_2$
$\tau_2 - \tau_3$
$\tau_3 - \tau_4$
τ_4
$B(q)\ddot{\bar{q}} + C(q, \dot{q})\dot{\bar{q}} + G(q)$



Theorem 1. *The continuously differentiable trajectory* $\bar{q}(t)$ *is a globally asymptotically stable equilibrium for the closed loop system (28) under the hypotheses of Corollary 1.*

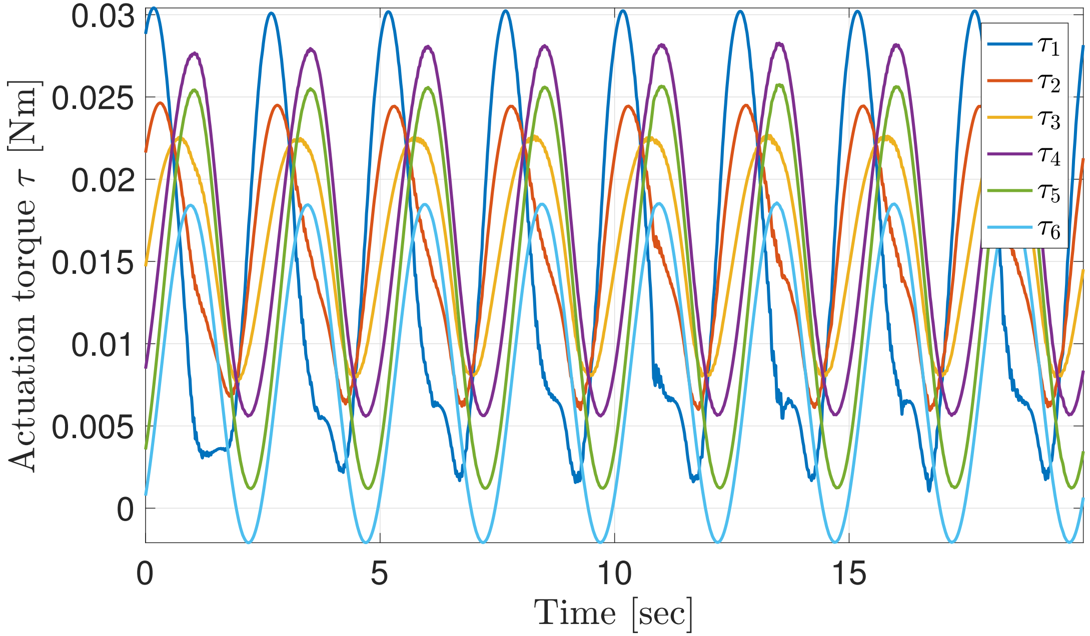
Della Santina C., et al. "Model-based dynamic feedback control of a planar soft robot: trajectory tracking and interaction with the environment." IJRR

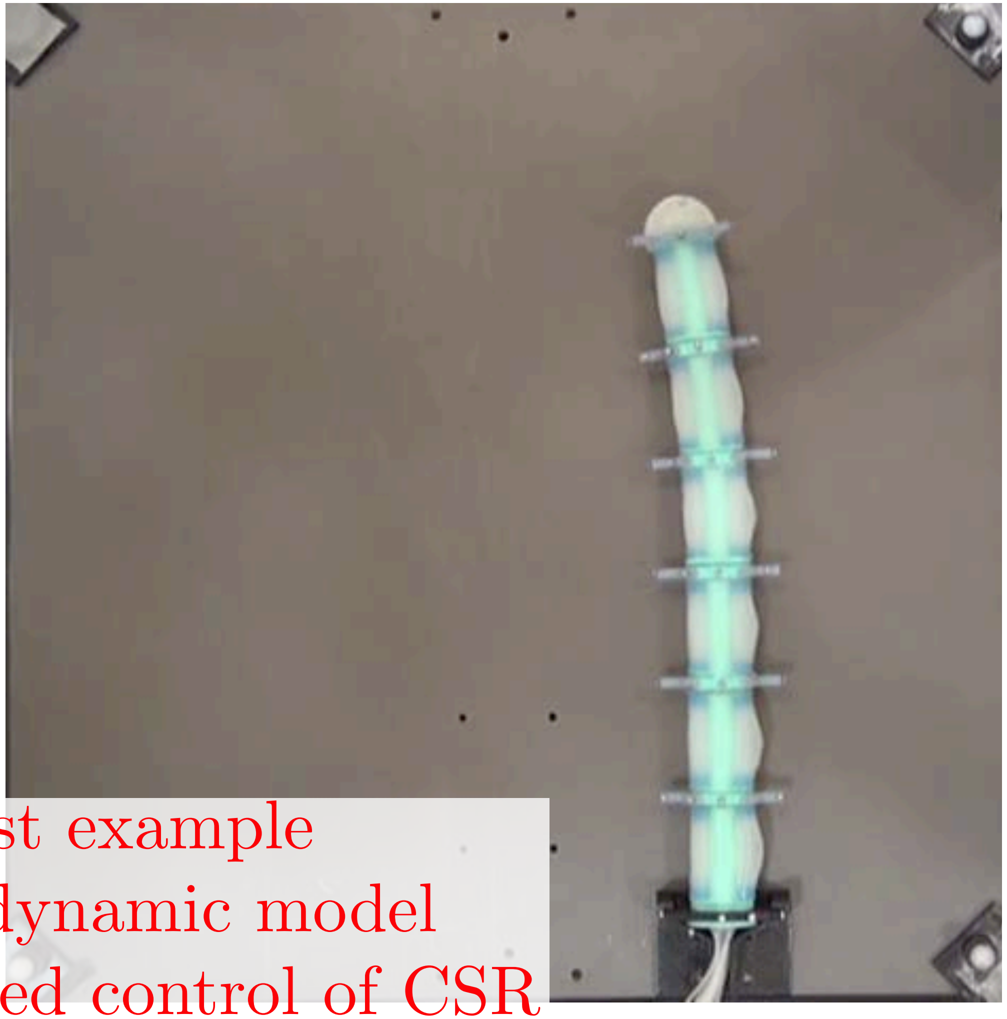


$$\bar{q}_i = 13^\circ\left(1 + \cos\left(\frac{4\pi}{5}\frac{\text{rad}}{\text{s}}t\right)\right)$$

$$\forall i \in \{1, \ldots, 6\}, \quad t \in [0, 20)\text{s}$$

Reference



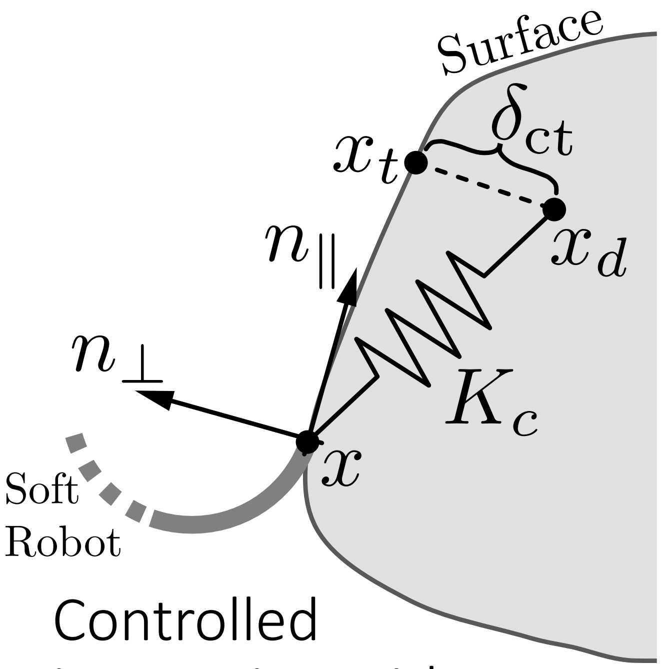


First example of dynamic model based control of CSR

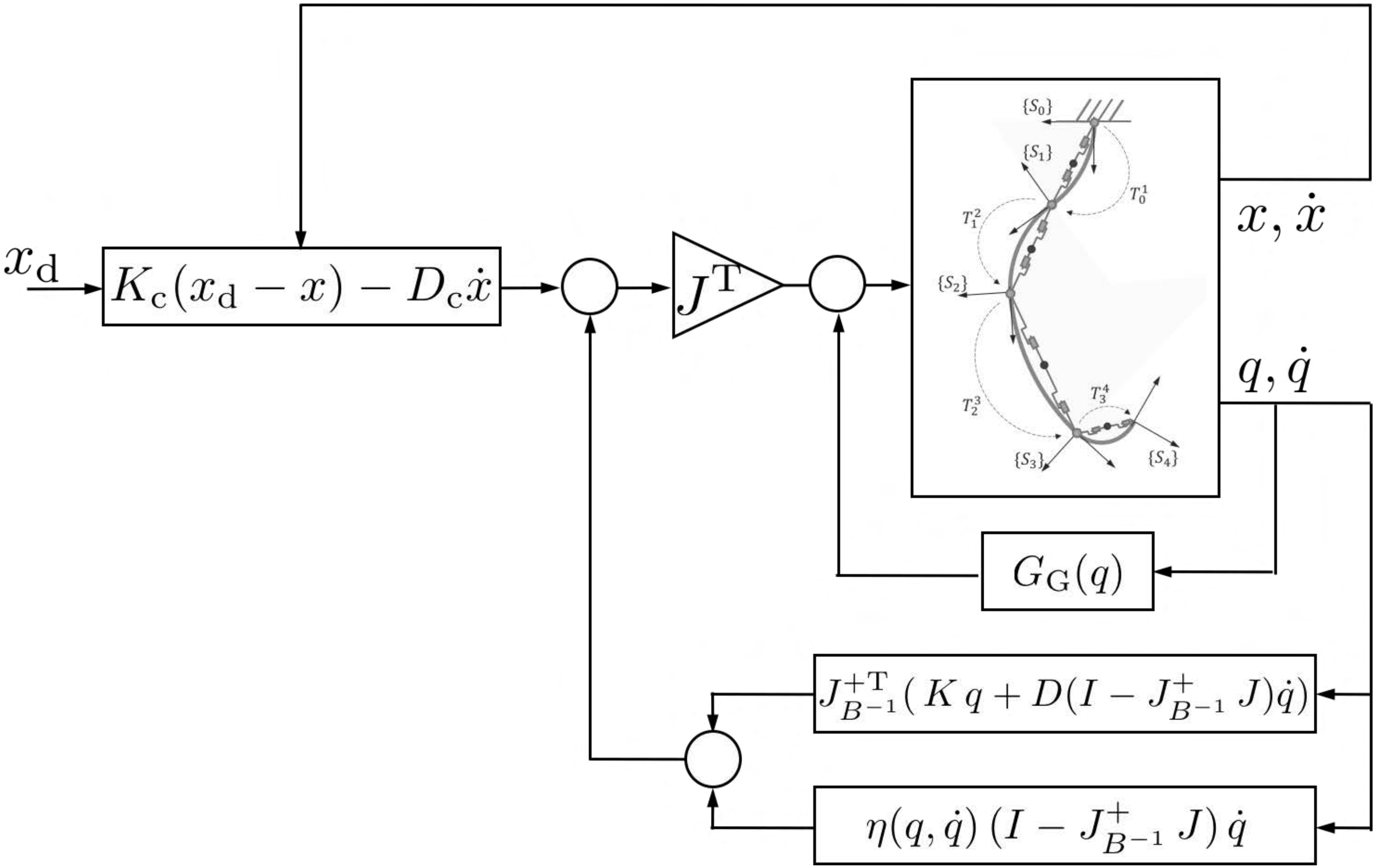
Della Santina C., et al. "Model-based dynamic feedback control of a planar soft robot: trajectory tracking and interaction with the environment." IJRR

Della Santina C., et al. "Dynamic control of soft robots interacting with the environment." *! "#"$"%&*18
Della Santina C., et al. "Model-based dynamic feedback control of a planar soft robot: trajectory tracking and interaction with the environment." IJRR

Some Cartesian impedance control too…

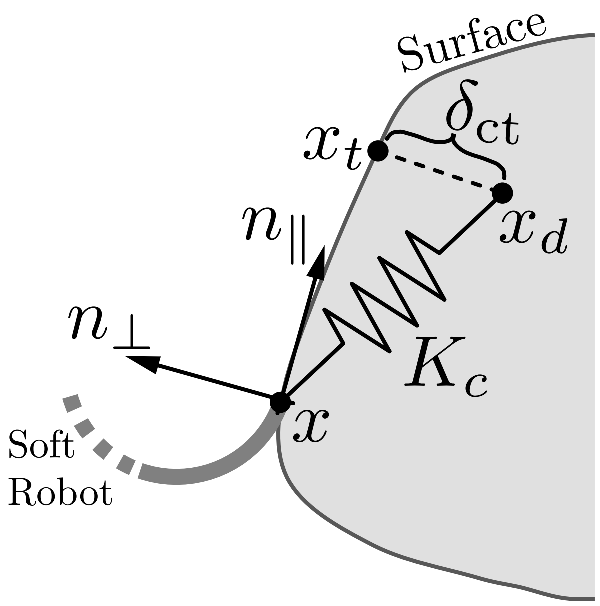


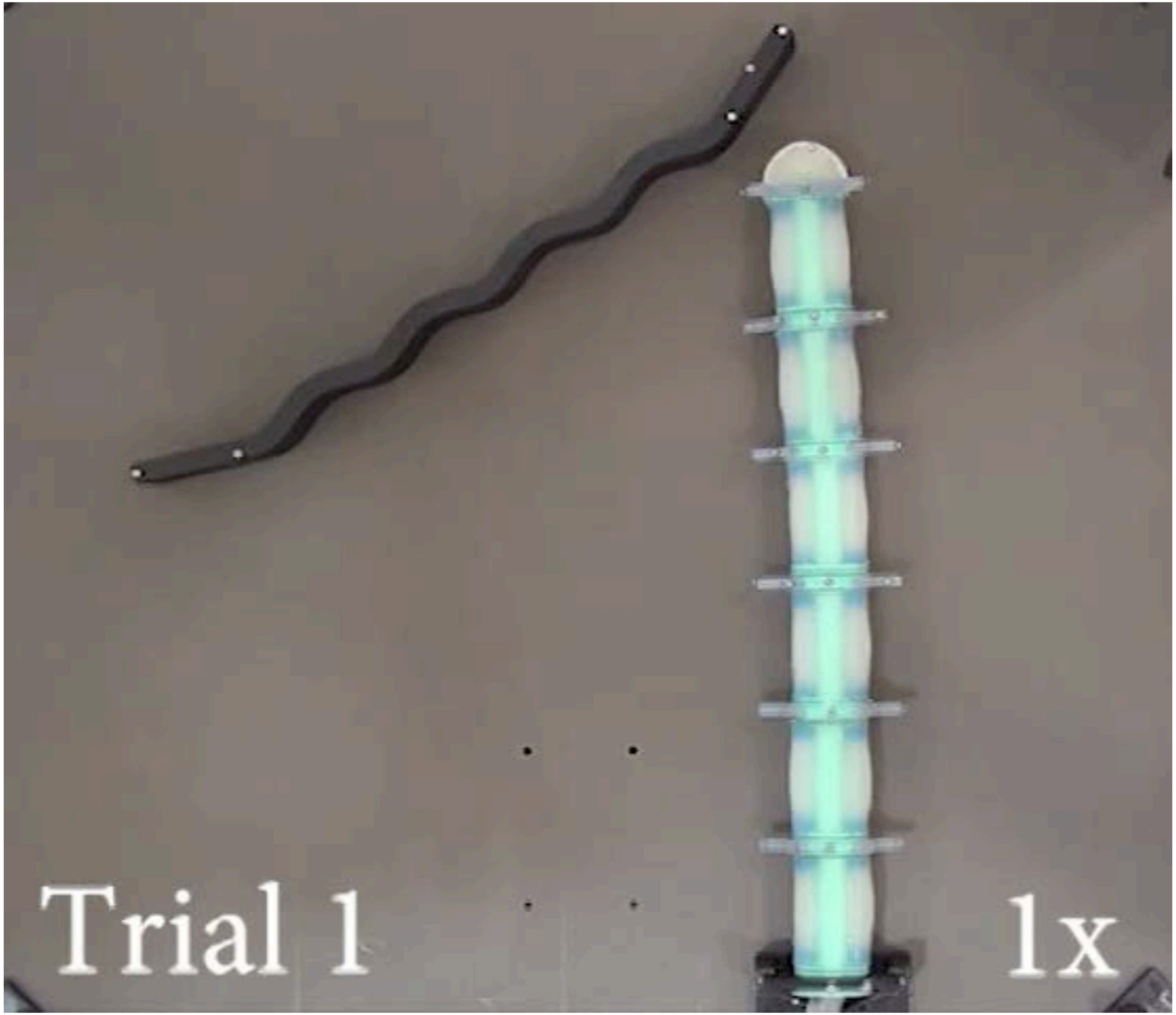
Controlled interaction with the environment

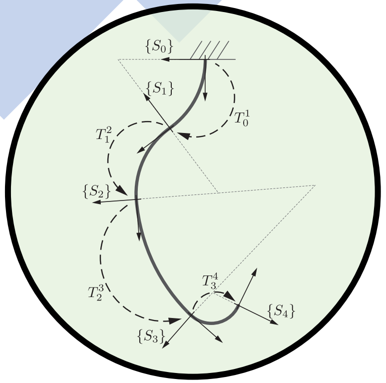


Some Cartesian impedance control too…

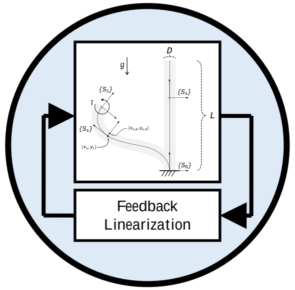
$$\tau = J^{\mathrm{T}}(q) J_B^{+\mathrm{T}}(q)(K\,q + D\dot{q}) + G(q)$$
$$+ J^{\mathrm{T}}(q)\,\eta(q,\dot{q})\,(I - J_B^{+}(q)\,J(q))\,\dot{q}$$
$$+ J^{\mathrm{T}}(q)(K_{\mathrm{c}}(x_{\mathrm{d}} - x) - D_{\mathrm{c}}J(q)\dot{q})\ ,$$







Feedback Model Based Control Is Robust to Rough Approximations



If You Want to Dig More Do That in a Control Oriented Way

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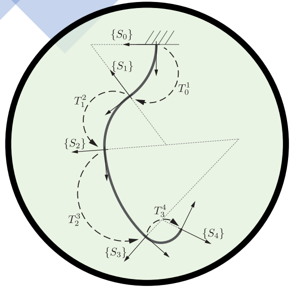

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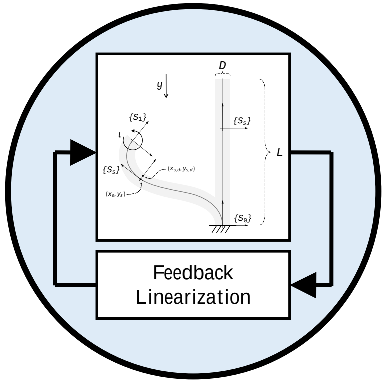

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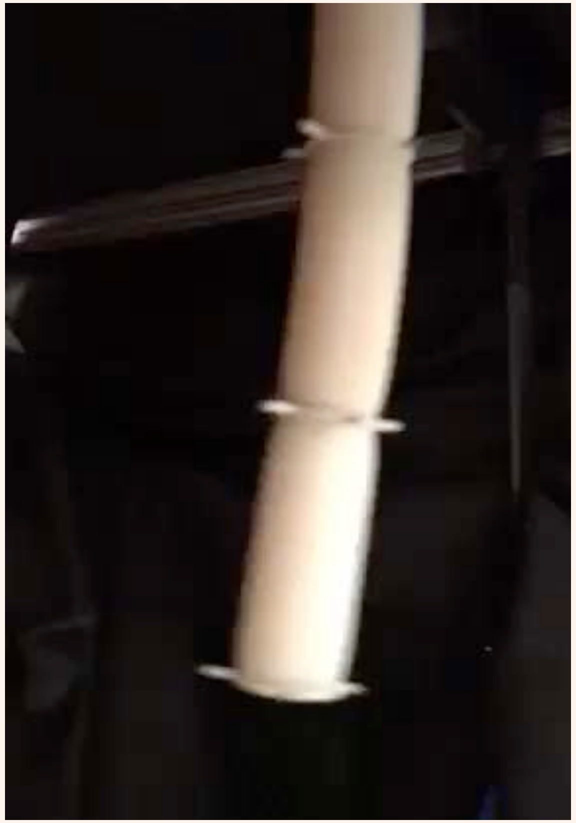
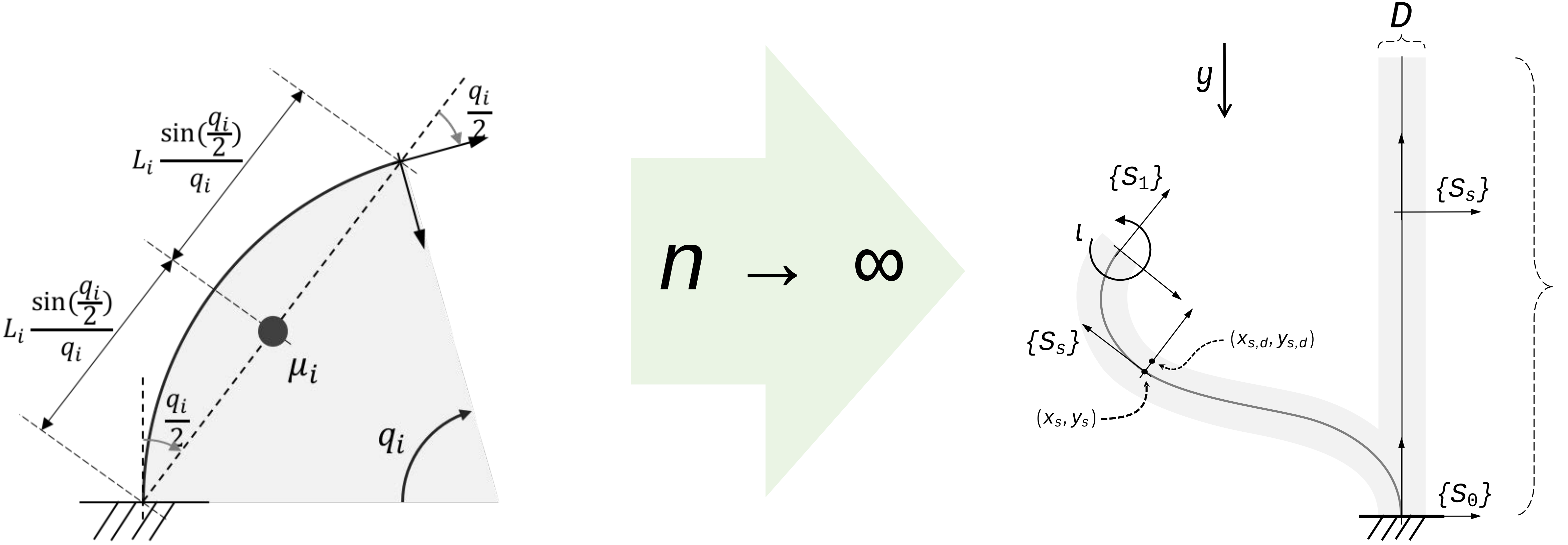


Reduced performance

Persistent oscillations

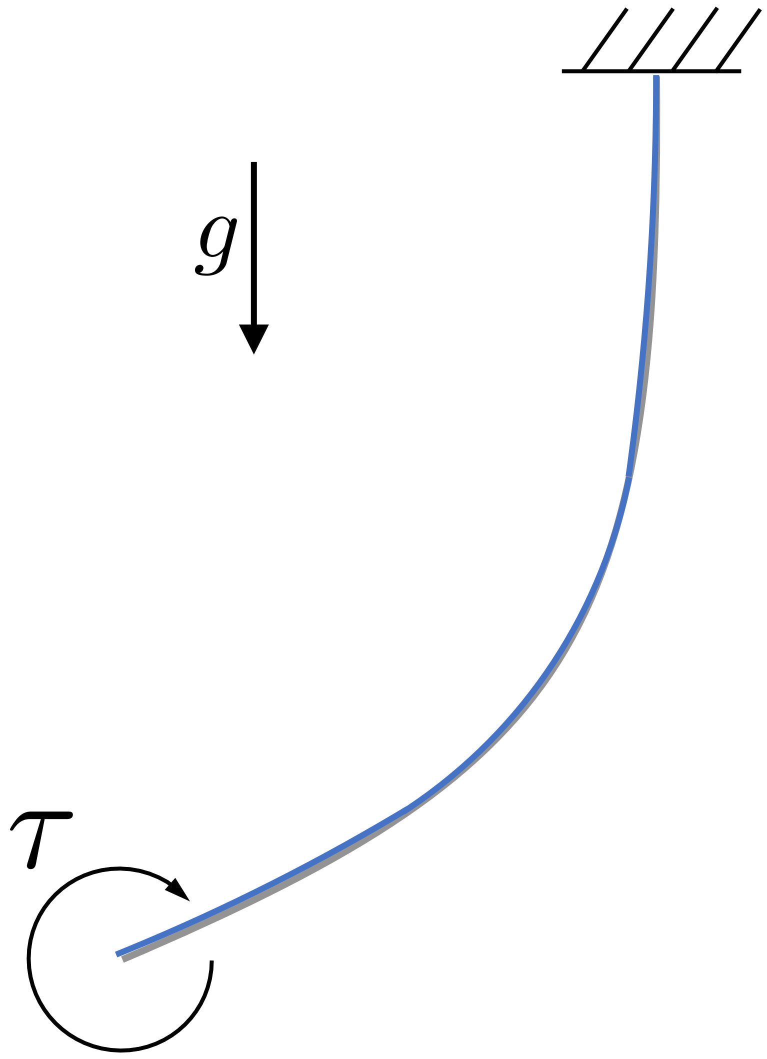
Why does it work?

What to correct it when it does not work?



To Infinity and Beyond

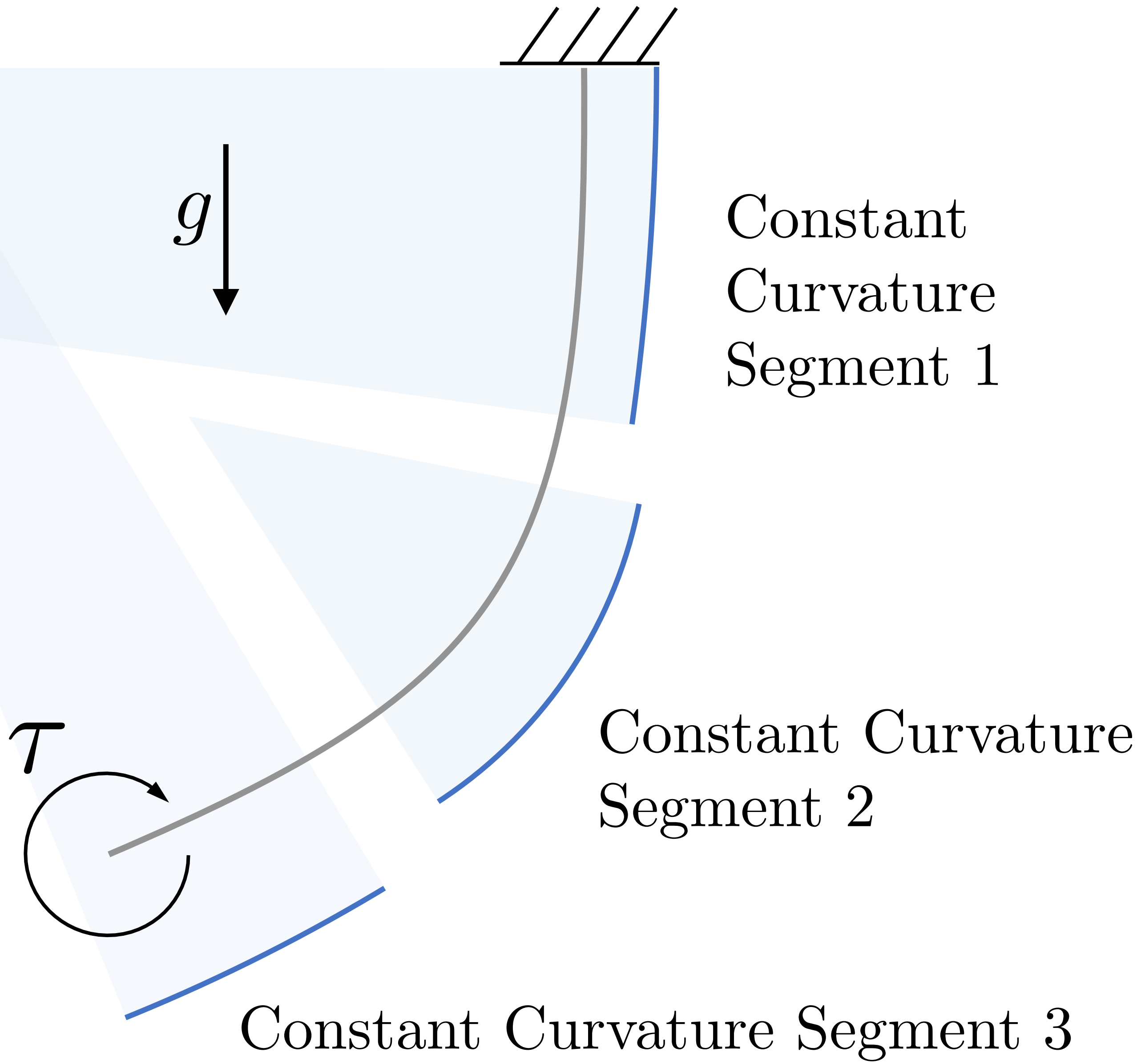
What to do?



Della Santina et al. "Exact task execution in highly under-actuated soft limbs: an operational space based approach” *RAL 2019*

What to do?

Just take a less coarse discretization!



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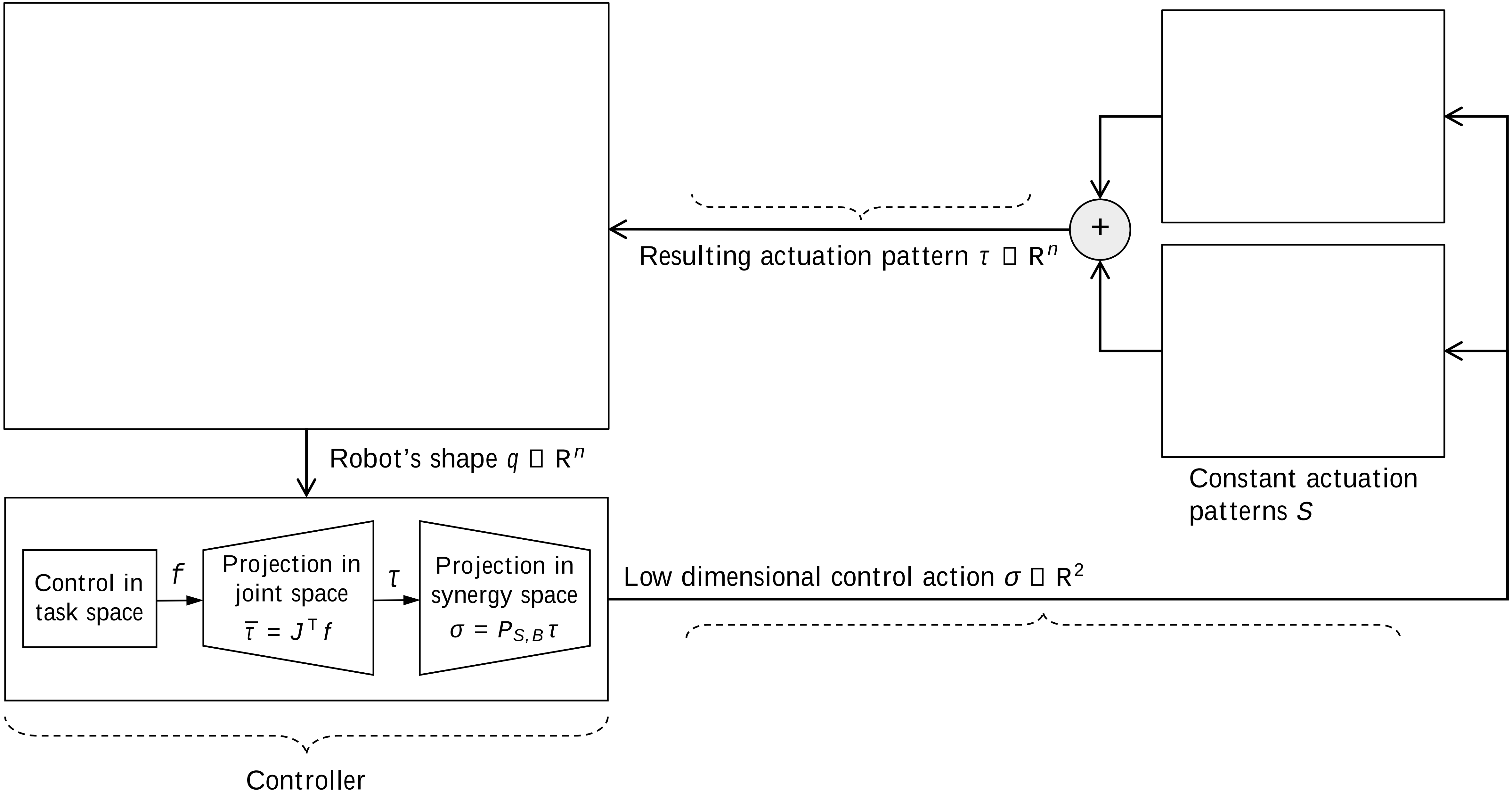

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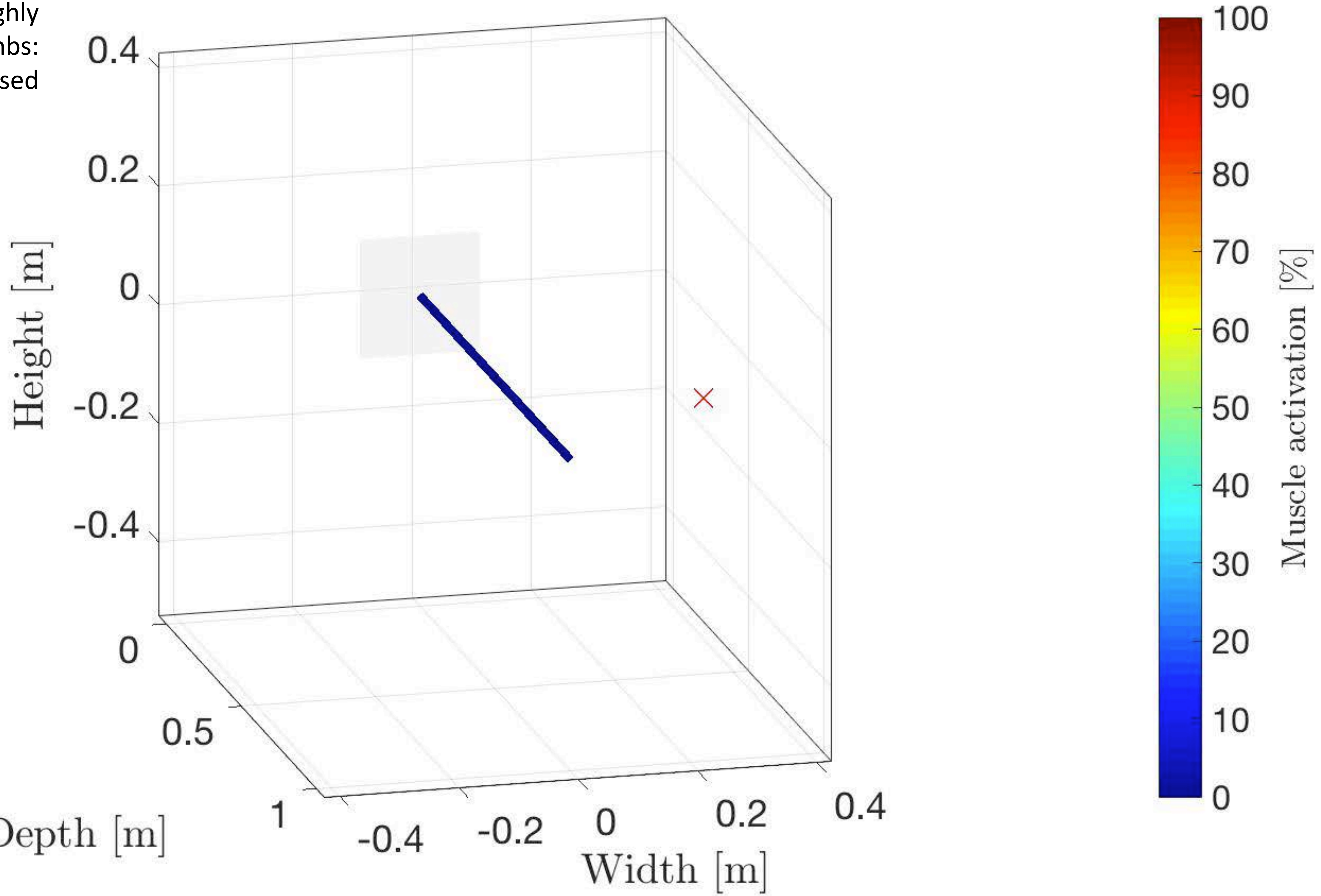
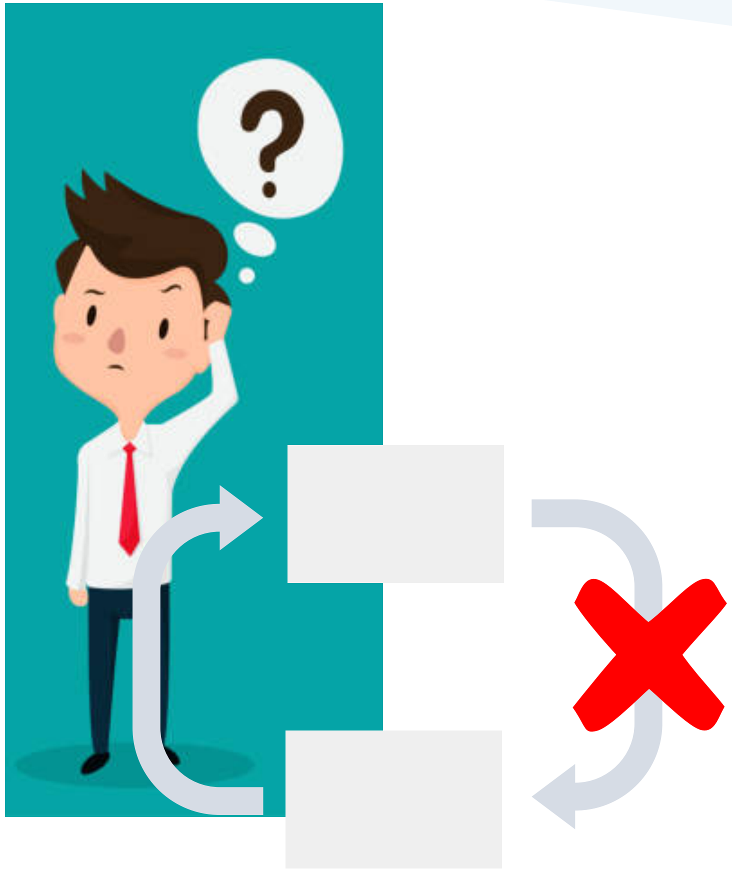
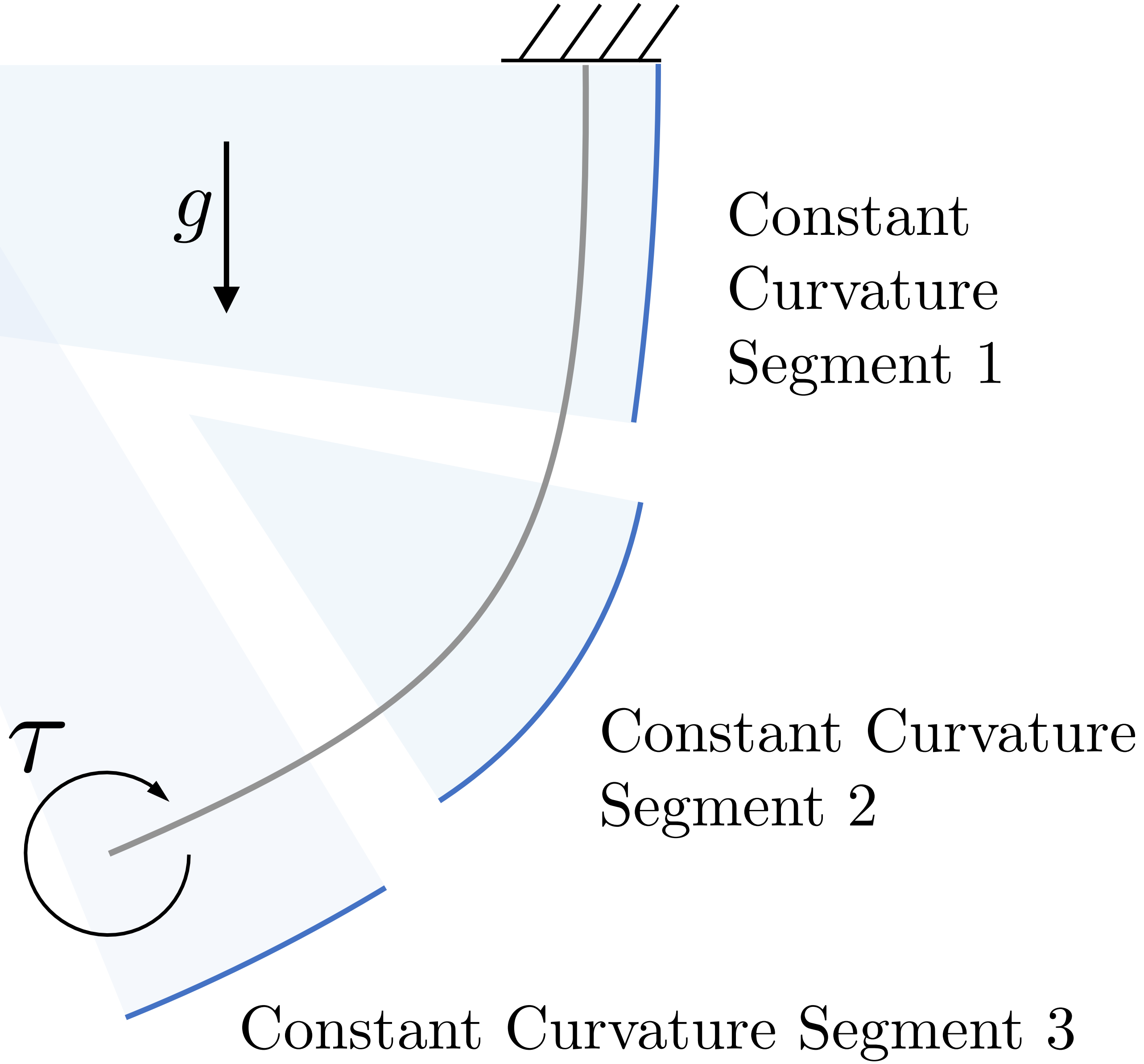


Just take a less coarse discretization!

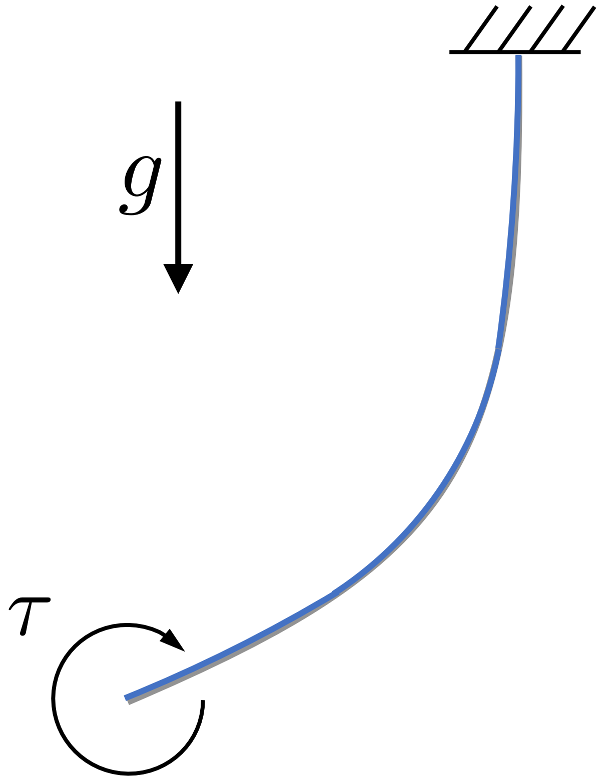




Della Santina et al. "Exact task execution in highly under-actuated soft limbs: an operational space based approach” *RAL 2019*

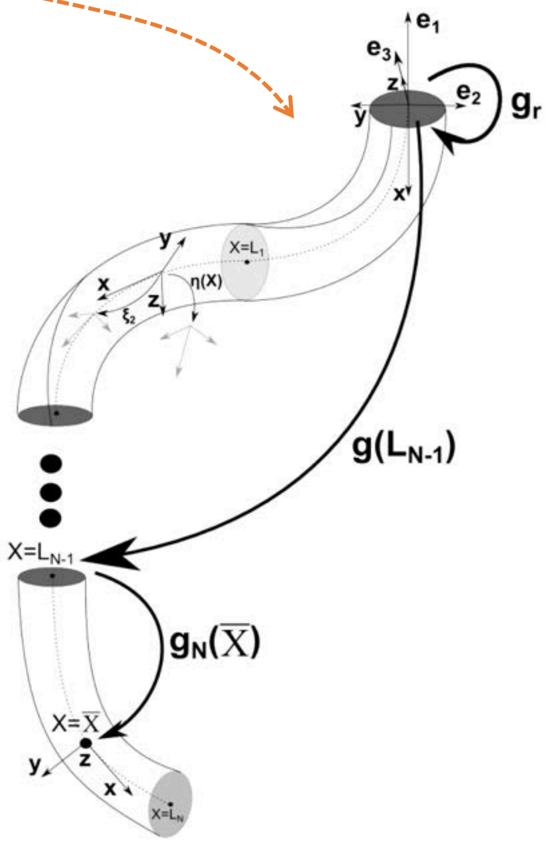
Just take a less coarse discretization!

Quite advanced!



Della Santina et al. "Exact task execution in highly under-actuated soft limbs: an operational space based approach" *RAL 2019*

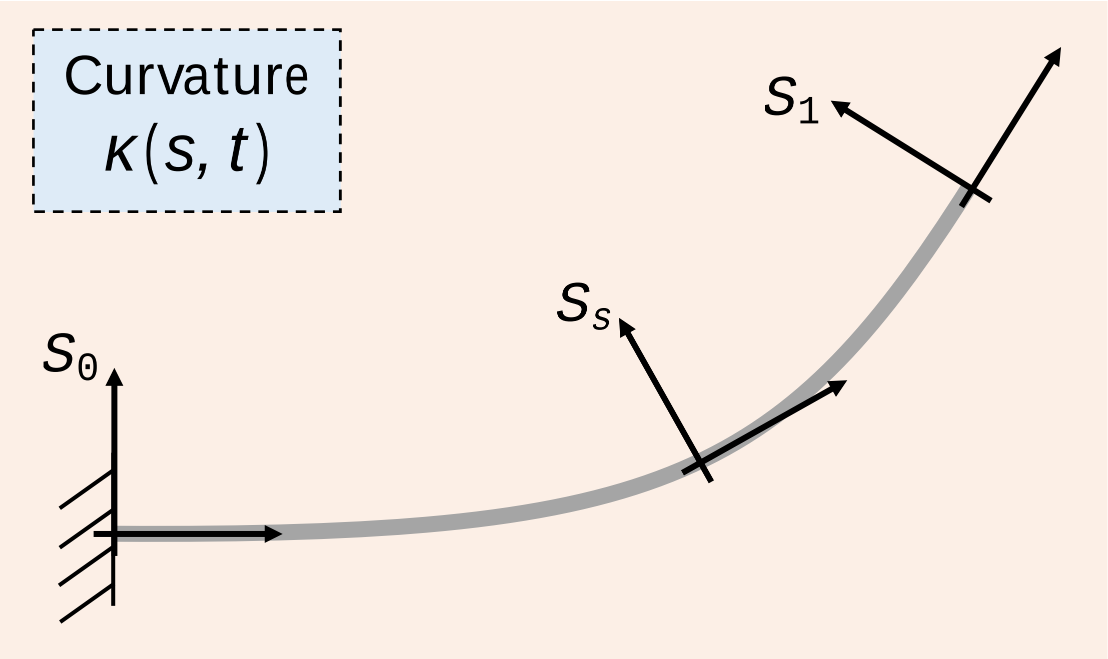
Coevoet et al. “Software toolkit for modeling, simulation and control of soft robots” Advanced Robotics 2017



Renda et al. “Discrete Cosserat Approach for Multi-Section Soft Robots Dynamics” TRO 2016

Kinematics

Configuration space

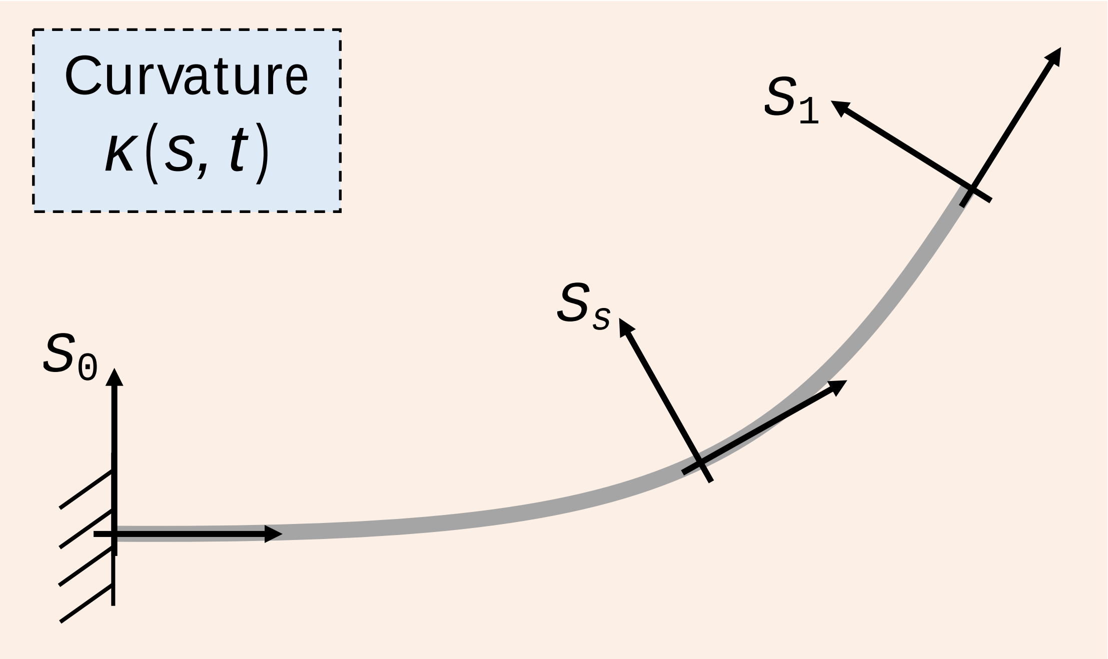


Della Santina, Cosimo, and Daniela Rus. "Control oriented modeling of soft robots: the polynomial curvature case." *RAL* (2019)

Kinematics

Configuration space

Hypothesis:
κ is analytical in s



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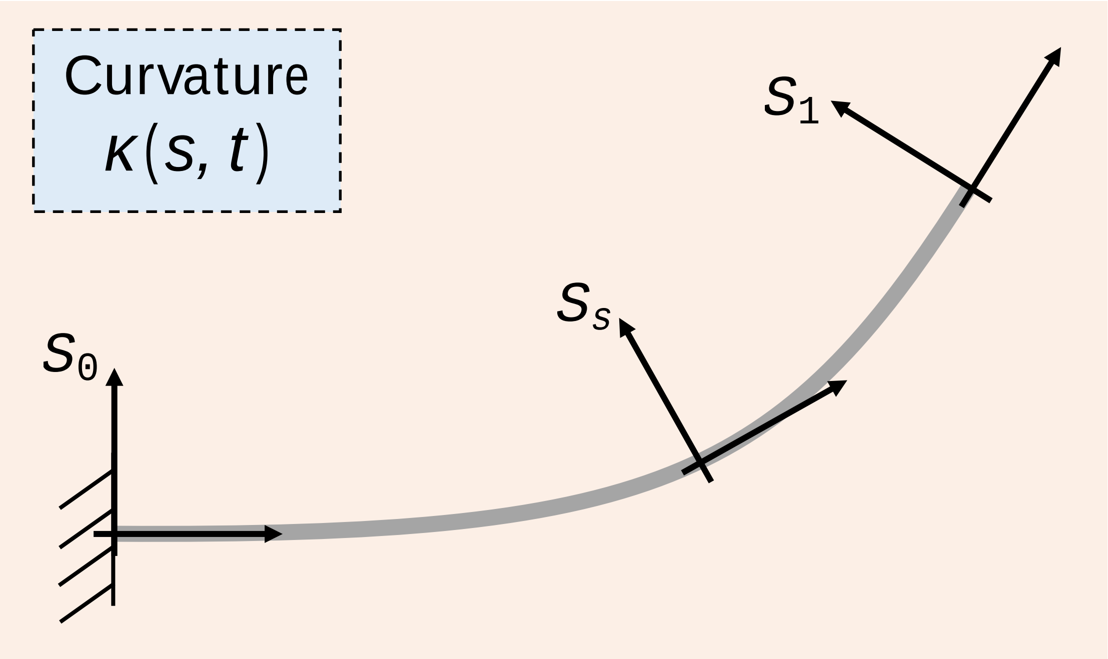
Kinematics

Configuration space



Hypothesis:
κ is analytical in s

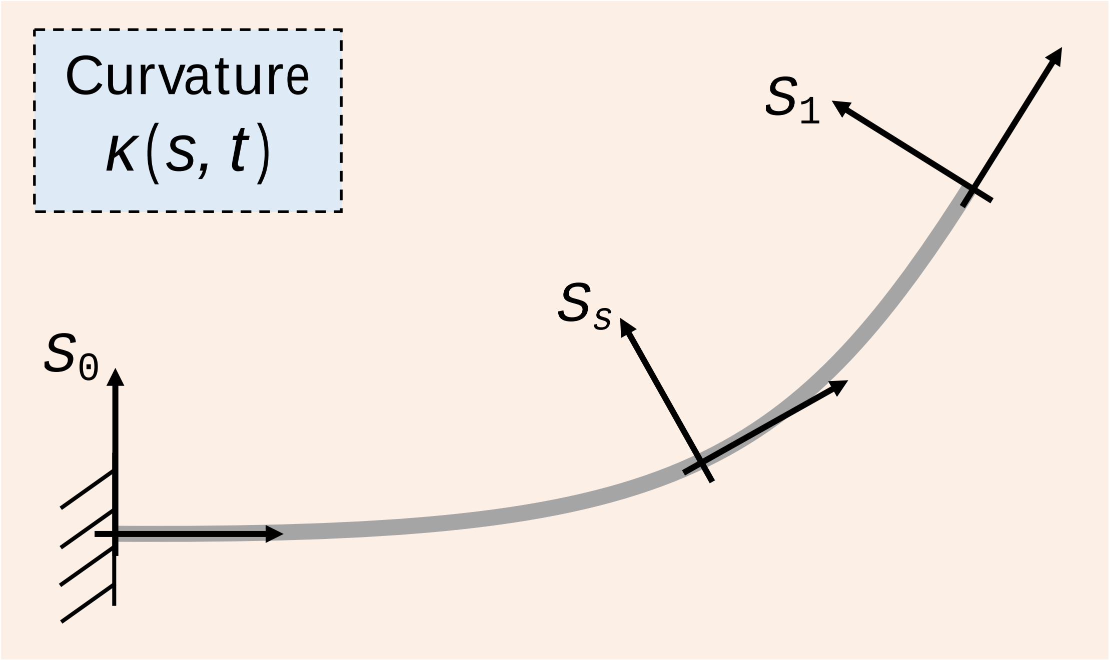
⊏

$$\kappa(s, t) = \sum_{i=0}^{\infty} \theta_i(t) s^i$$

Della Santina, Cosimo, and Daniela Rus. "Control oriented modeling of soft robots: the polynomial curvature case." *RAL* (2019)

Kinematics

Configuration space



Hypothesis:
κ is analytical in s

⊏

Polynomial Curvature!!!

$$\kappa(s, t) = \sum_{i=0}^{\infty} \theta_i(t) s^i$$

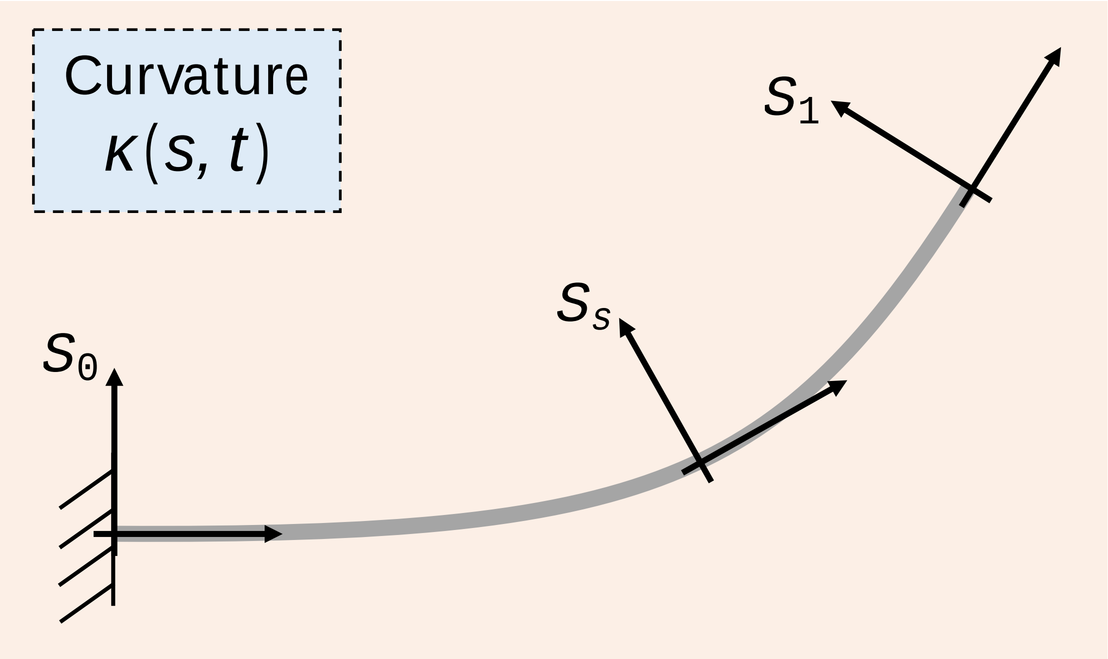
⊔

$$\kappa(s, t) \approx \sum_{0}^{m} \theta_i(t) s^i$$

Della Santina, Cosimo, and Daniela Rus. "Control oriented modeling of soft robots: the polynomial curvature case." *RAL* (2019)

Kinematics

Configuration space

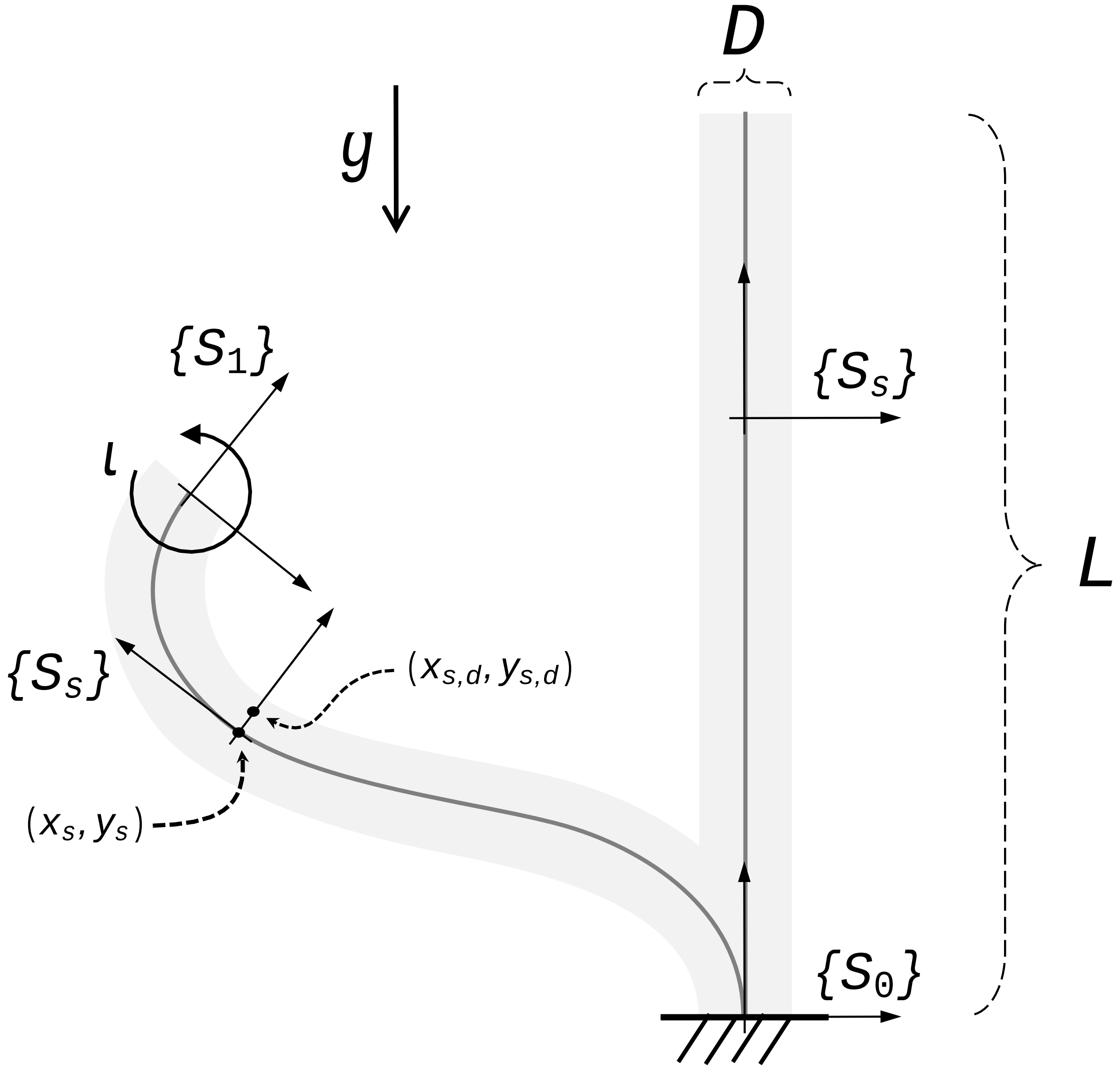


Hypothesis:
κ is analytical in s

$$\Downarrow$$

$$\kappa(s, t) = \sum_{i=0}^{\infty} \theta_i(t) s^i \quad \Downarrow \quad \kappa(s, t) \approx \sum_{0}^{m} \theta_i(t) s^i$$

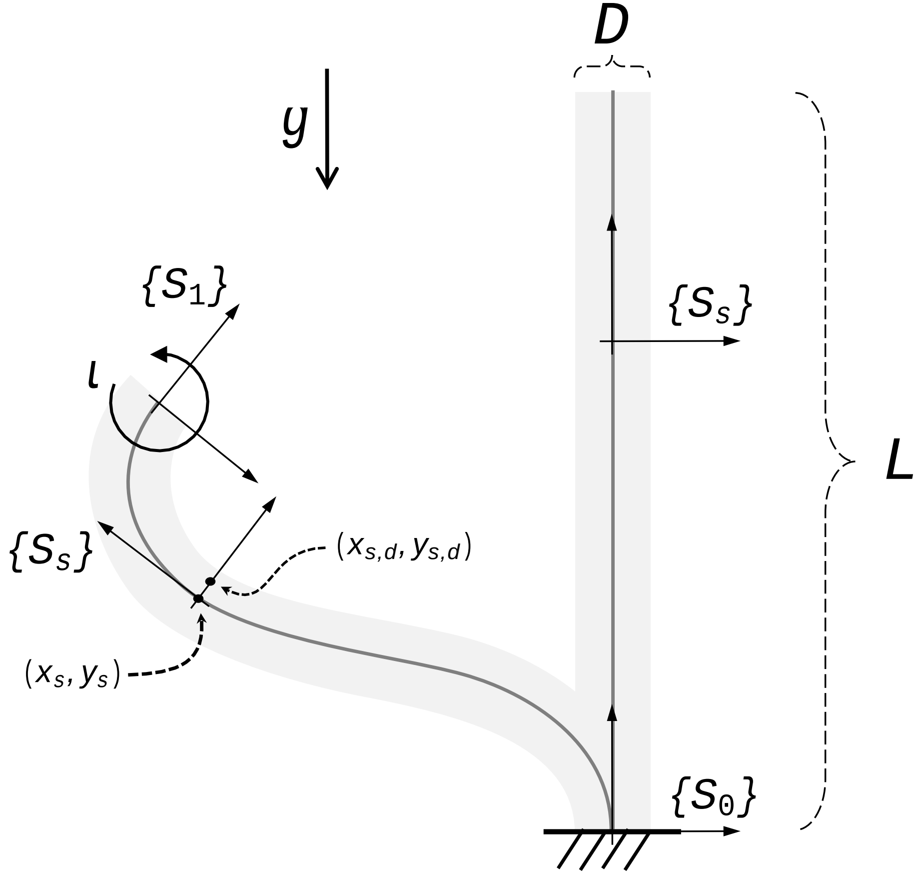
$$\theta = \left[\theta_0 \quad \dots \quad \theta_m\right]^T$$



Soft Inverted Pendulum

Della Santina, Cosimo. "The Soft Inverted Pendulum with Affine Curvature." *CDC* 2020

$$\tilde{B}(q)\ddot{q} + \tilde{C}(q,\dot{q})\dot{q} + \tilde{G}(q) + \tilde{K}q + \tilde{D}\dot{q} = \begin{bmatrix} \tau \\ 0 \end{bmatrix}$$



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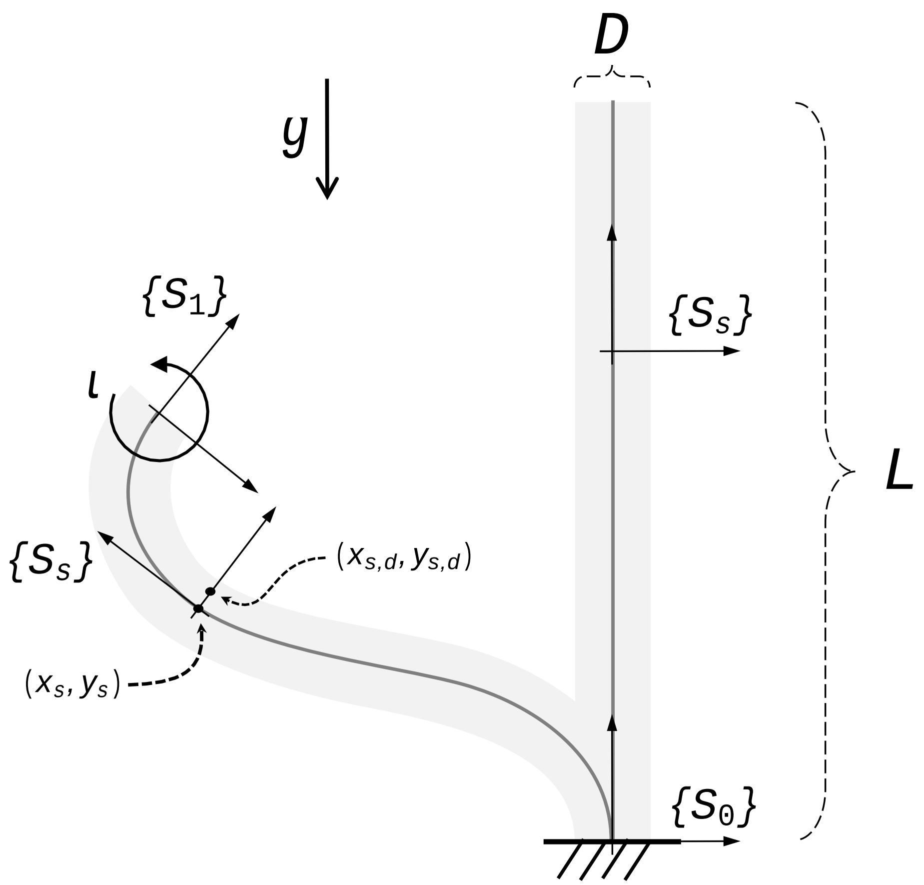
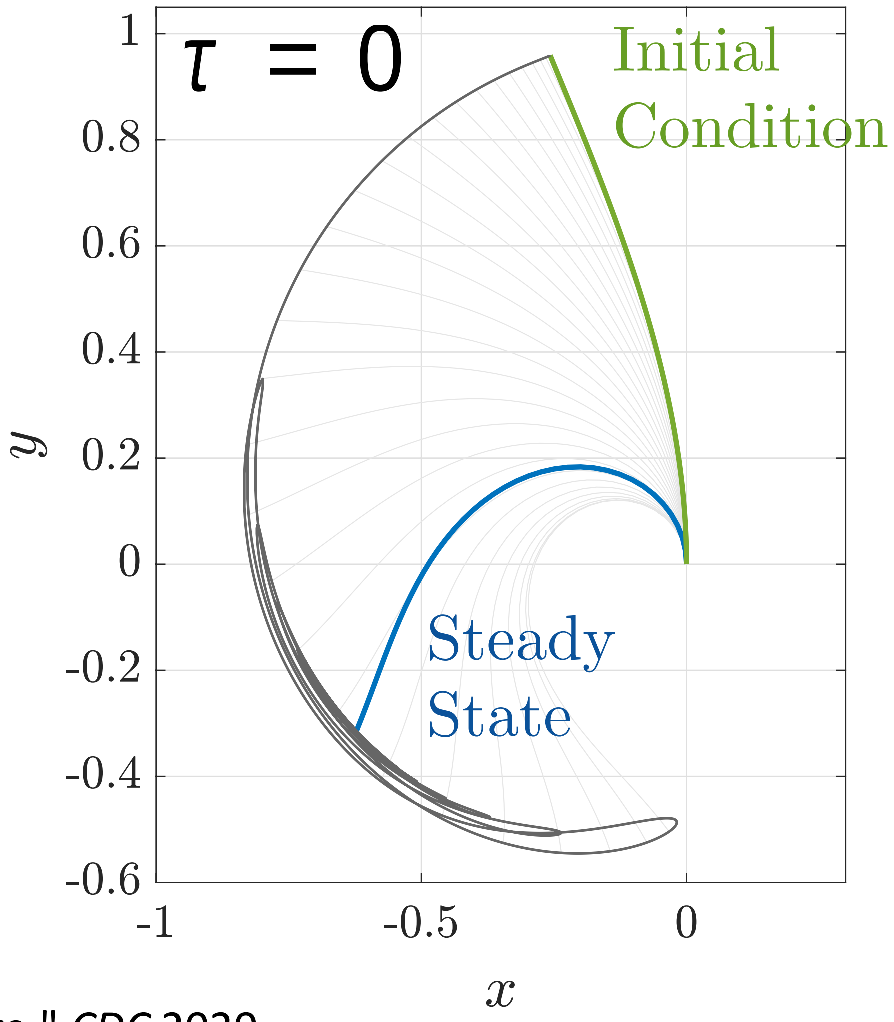


Soft Inverted Pendulum



Della Santina, Cosimo. "The Soft Inverted Pendulum with Affine Curvature." *CDC* 2020

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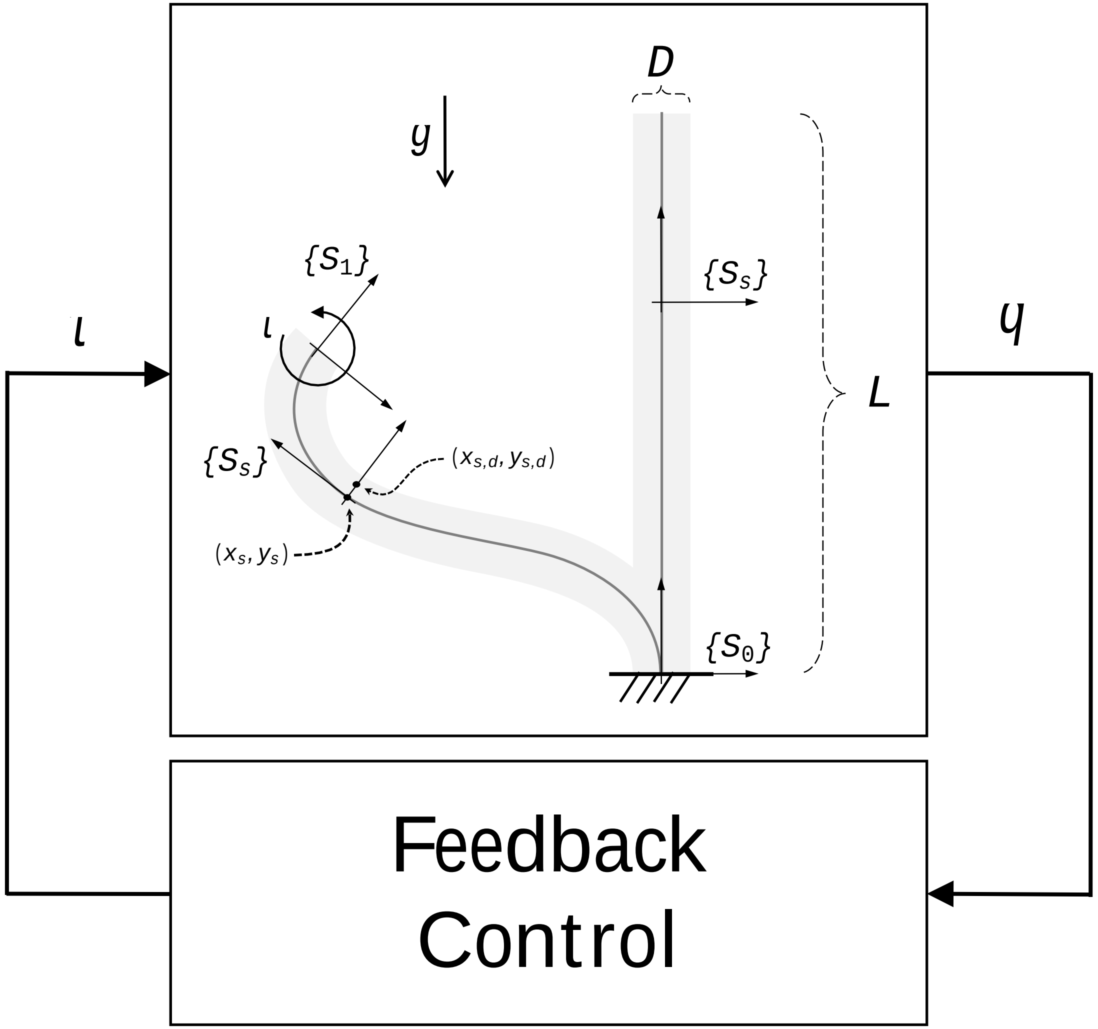

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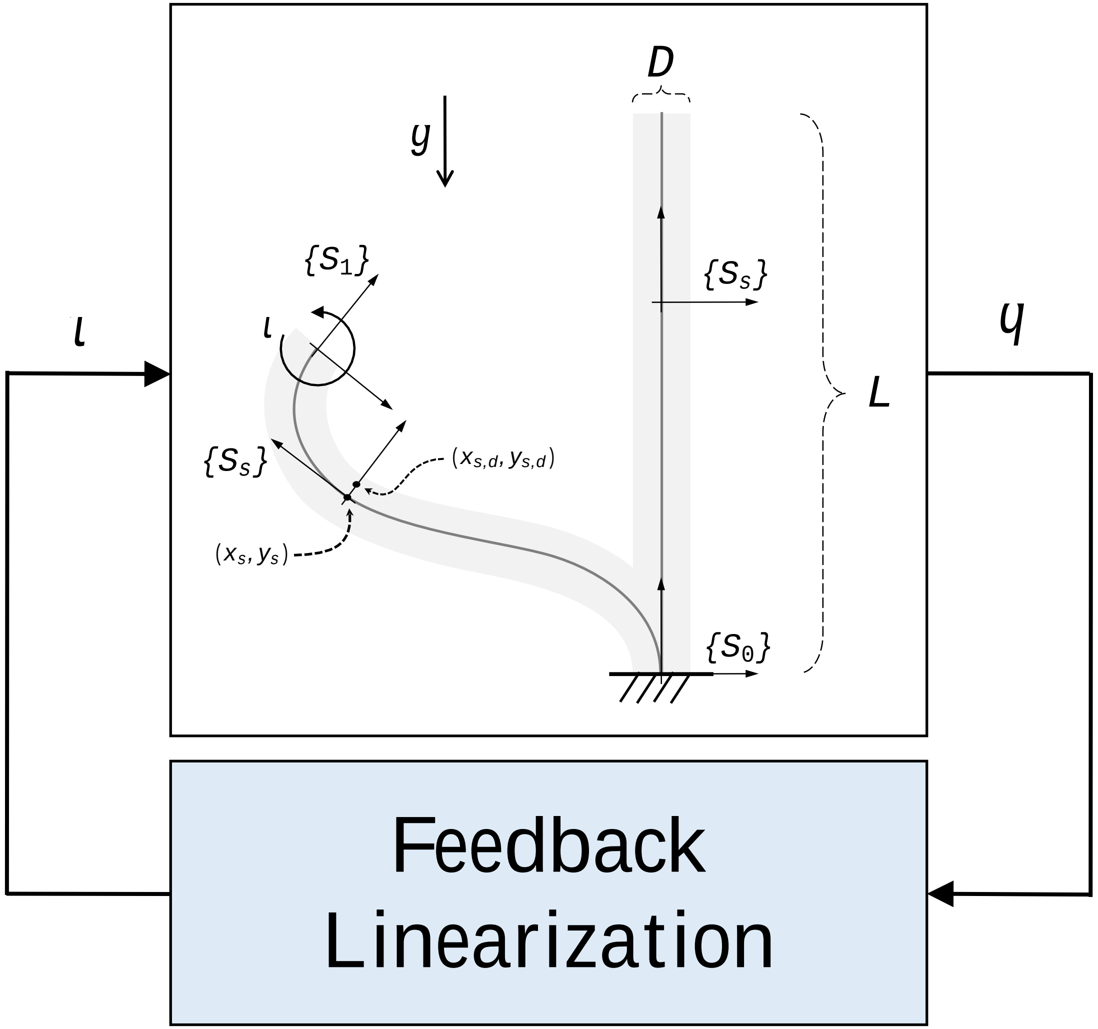
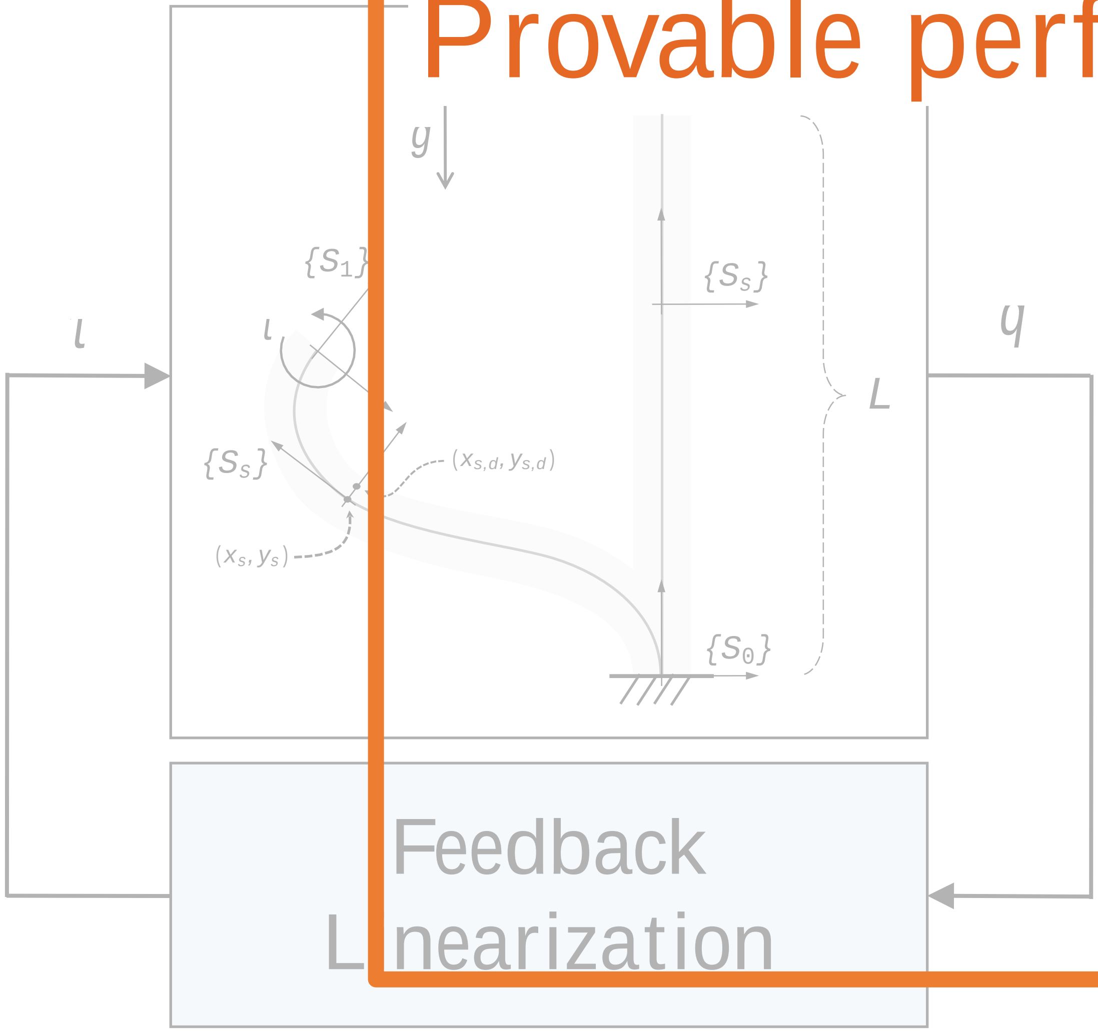


$$\tau = \left[h_1 - (\tilde{B}_{2,1}/\tilde{B}_{2,2})h_2\right] - \left(\tilde{B}_{1,1} - \tilde{B}^2_{2,1}/\tilde{B}_{2,2}\right)(\phi_P q_0 + \phi_D \dot{q}_0)$$

Della Santina, Cosimo. "The Soft Inverted Pendulum with Affine Curvature." *CDC* 2020

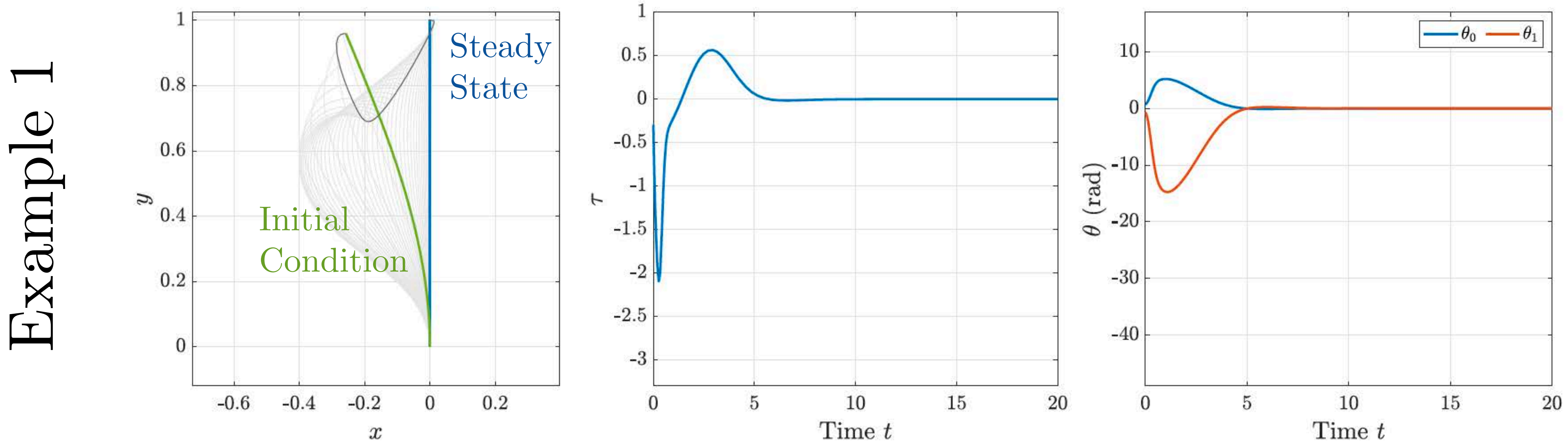
$$\tilde{B}(q)\ddot{q} + \tilde{C}(q,\dot{q})\dot{q} + \tilde{G}(q) + \tilde{K}q + \tilde{D}\dot{q} = \begin{bmatrix}\tau\\0\end{bmatrix}$$

Provable performance!

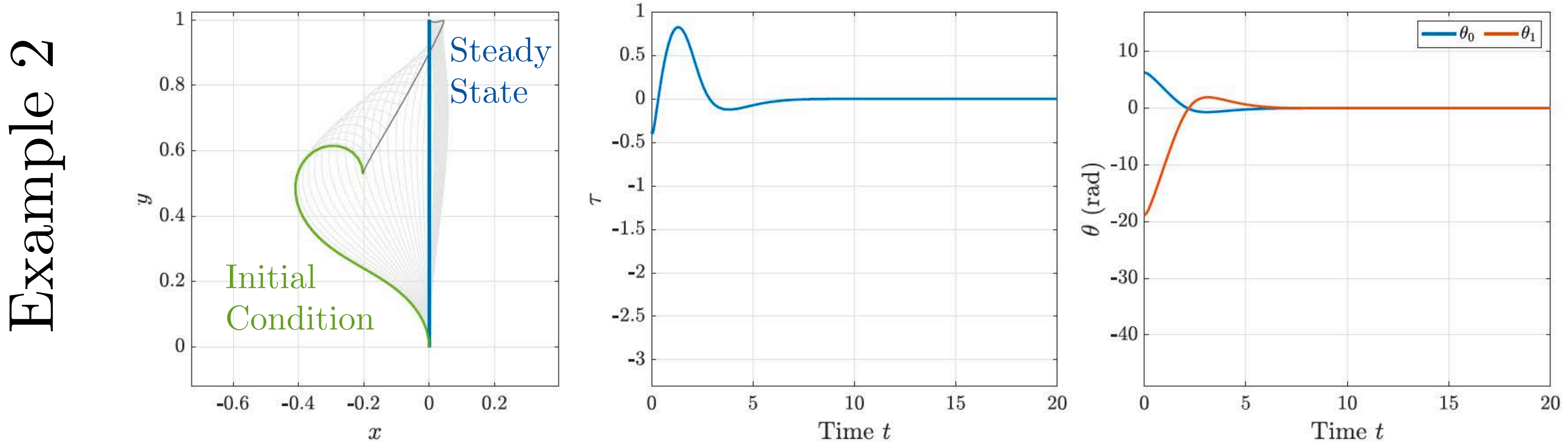


$$\tau = \left[h_1 - (\tilde{B}_{2,1}/\tilde{B}_{2,2})h_2\right] - \left[\tilde{B}_{1,1} - \tilde{B}_{2,1}^2/\tilde{B}_{2,2}\right](\phi_P q_0 + \phi_D \dot{q}_0)$$

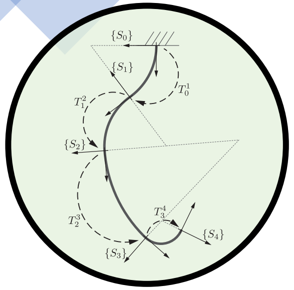
Della Santina, Cosimo. "The Soft Inverted Pendulum with Affine Curvature." *CDC* 2020



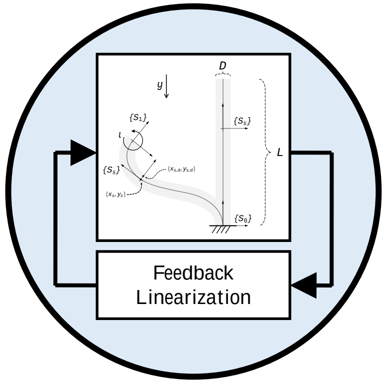
(a) Pendulum, $k = 0.25$ (b) Control action, $k = 0.25$ (c) Natural config., $k = 0.25$



(e) Pendulum, $k = 0.25$, far (f) Control action, $k = 0.25$, far (g) Natural config., $k = 0.25$, far

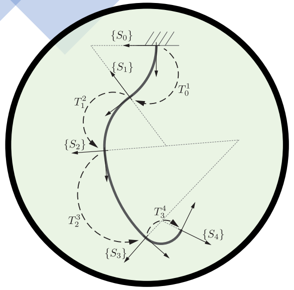


Feedback Model Based Control
Is Robust to Rough Approximations

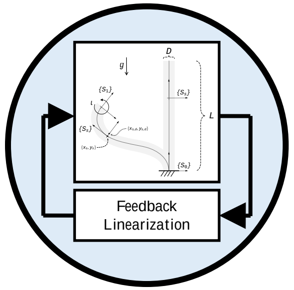


If You Want to Dig More
Do That in a Control Oriented Way

If You Want to Stick to the Simple Model,
Consired Control-Driven Ways to Improve It



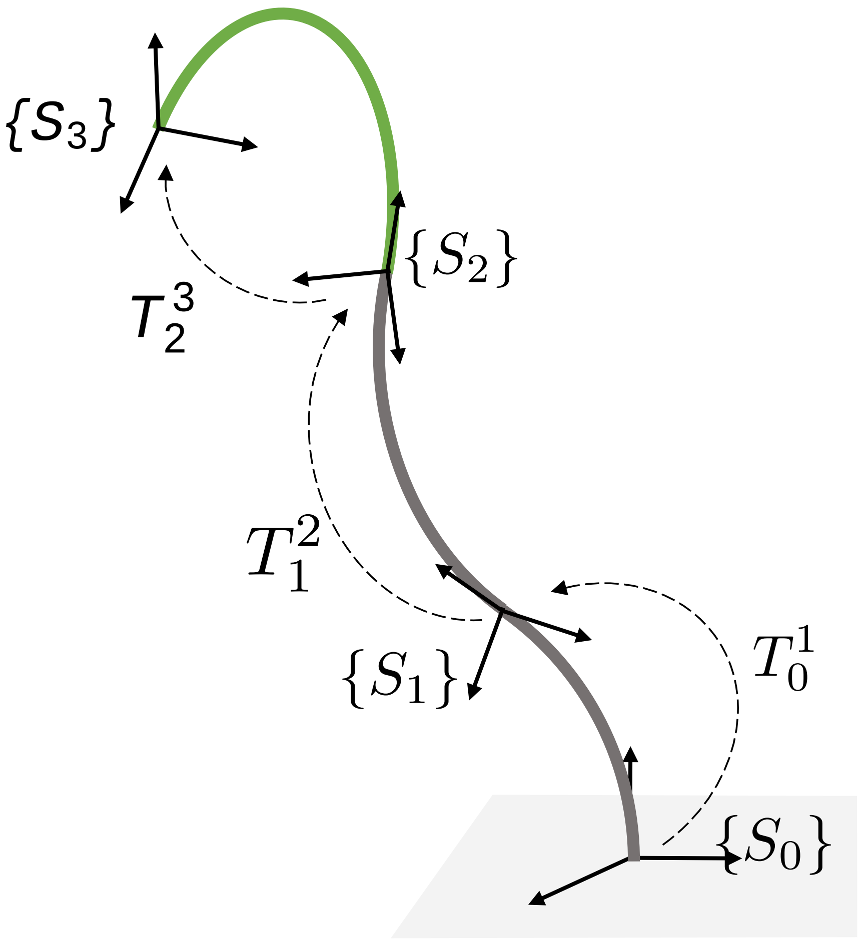
Feedback Model Based Control
Is Robust to Rough Approximations

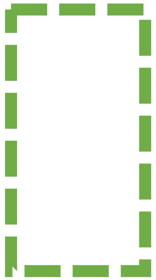


If You Want to Dig More
Do That in a Control Oriented Way

If You Want to Stick to the Simple Model,
Consired Control-Driven Ways to Improve It

Standard Extension of PCC to 3D (kinematics)





Della Santina, Cosimo, Antonio Bicchi, and Daniela Rus. "On an improved state parametrization for soft robots with piecewise constant curvature and its use in model based control." *IEEE Robotics and Automation Letters* 5.2 (2020): 1001-1008.

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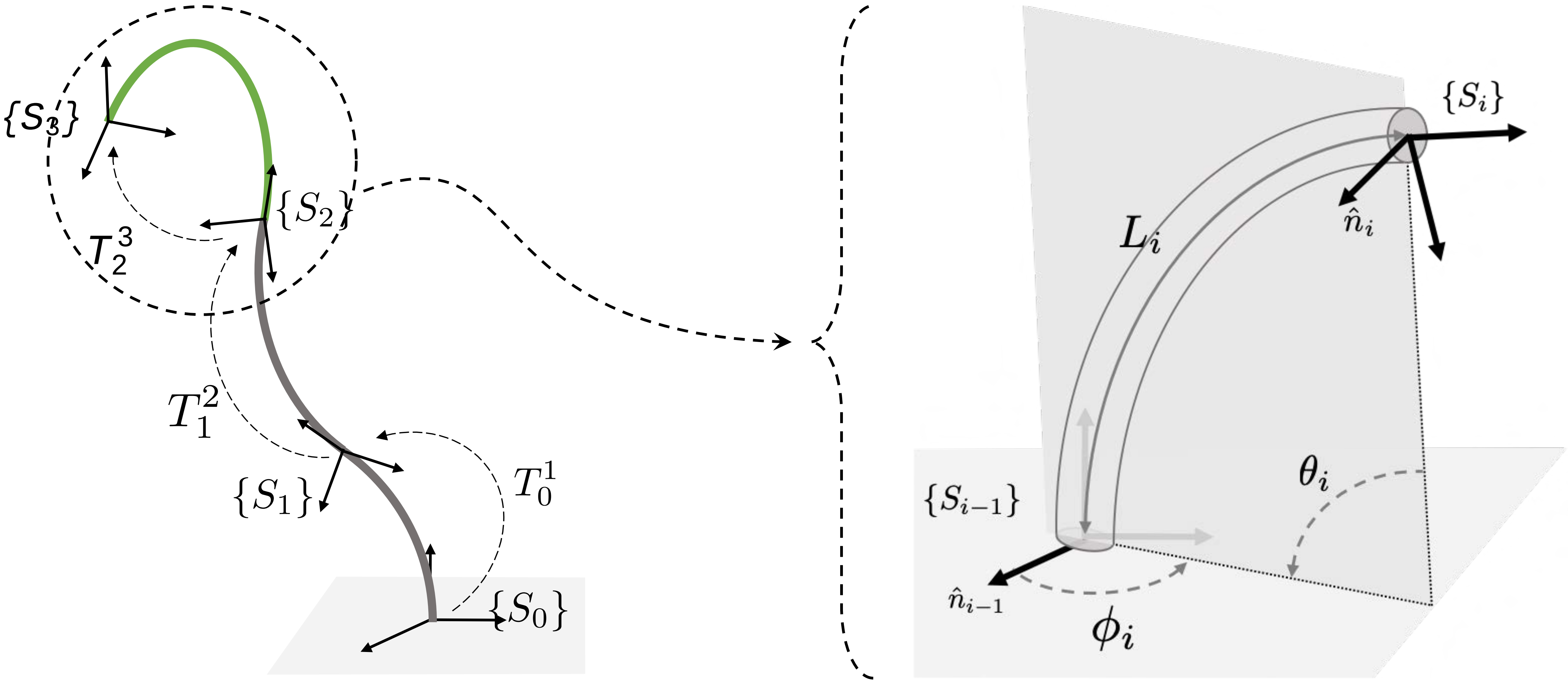

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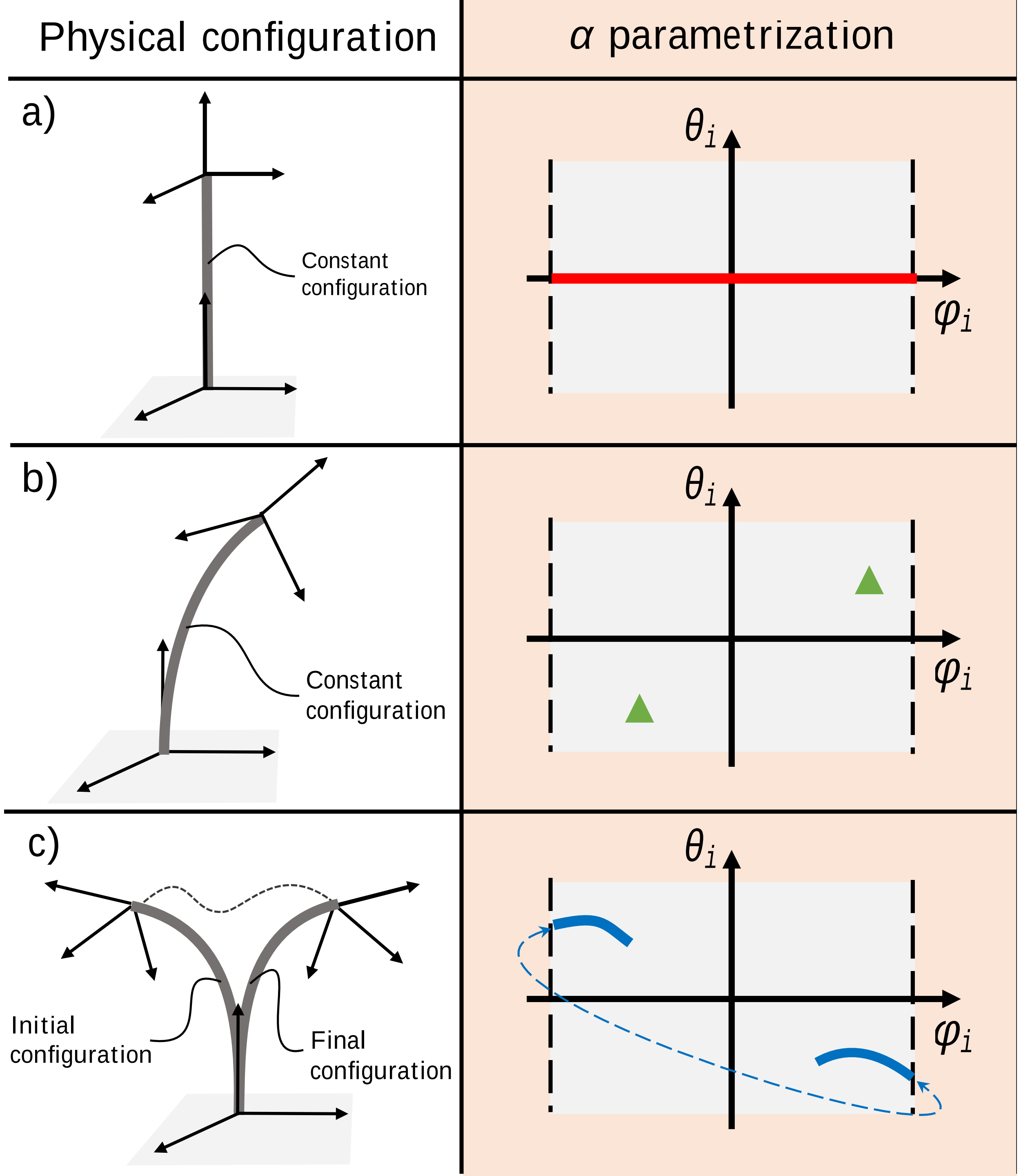
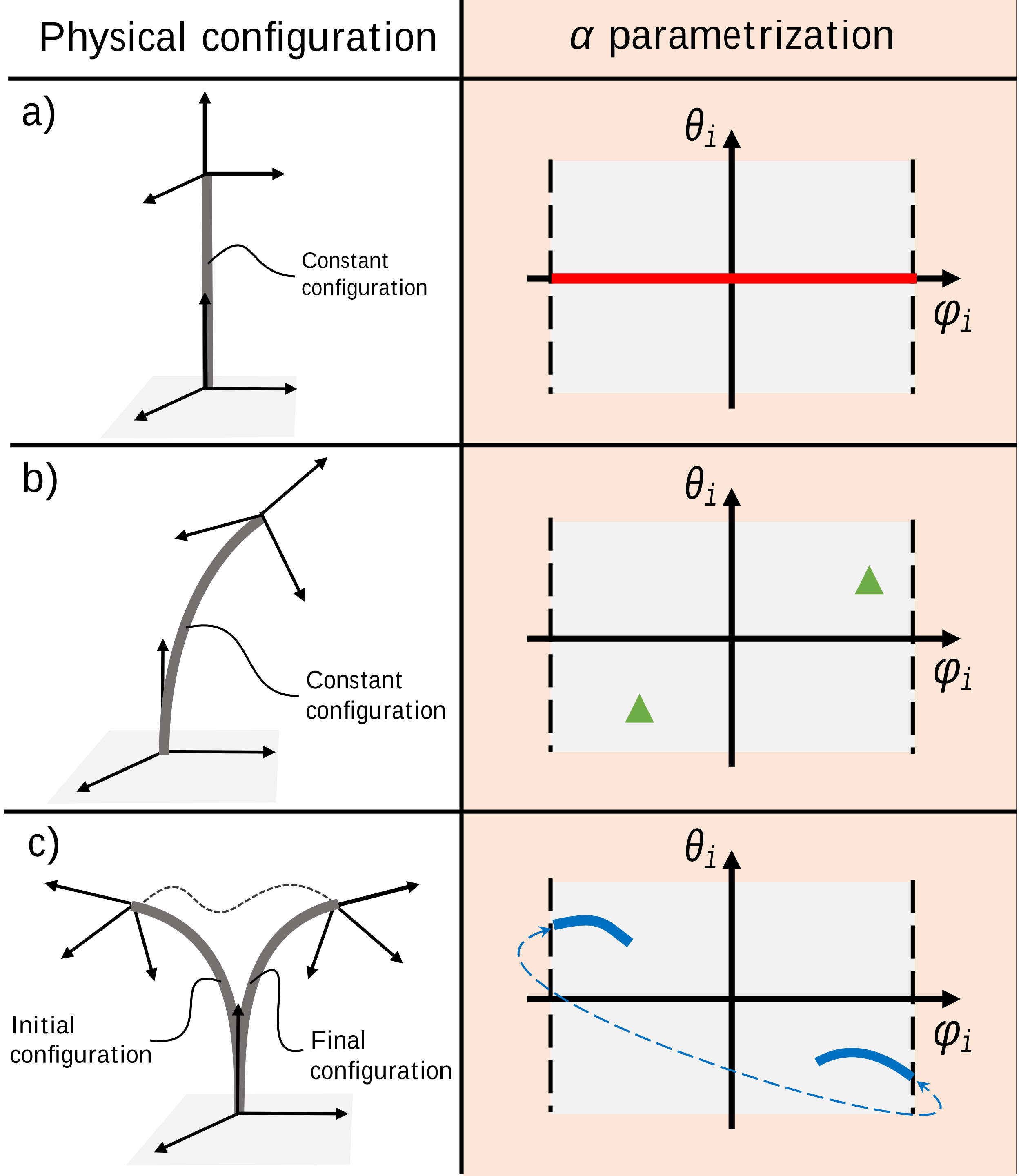
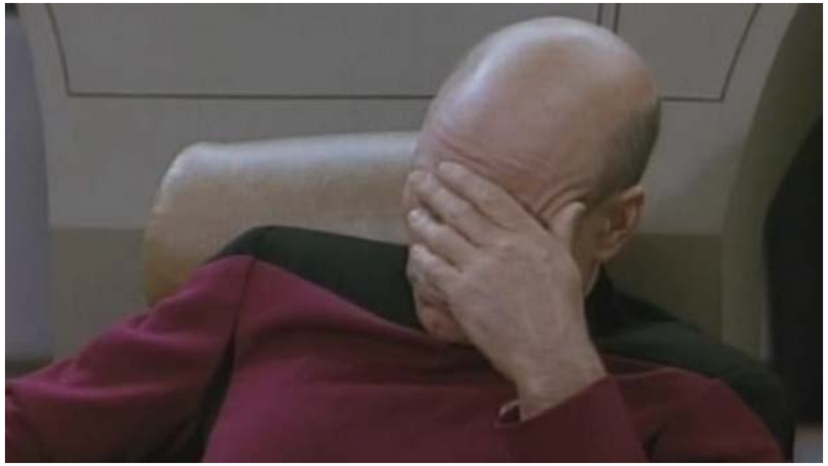

Physical configuration
α parametrization
a)
Constant
configuration
θi
φi
b)
Constant
configuration
θi
φi
c)
Initial
configuration
Final
configuration
θi
φi



Physical configuration
α parametrization
a)
Constant configuration
θi
φi
b)
Constant configuration
θi
φi
c)
Initial configuration
Final configuration
θi
φi

S^n Topology

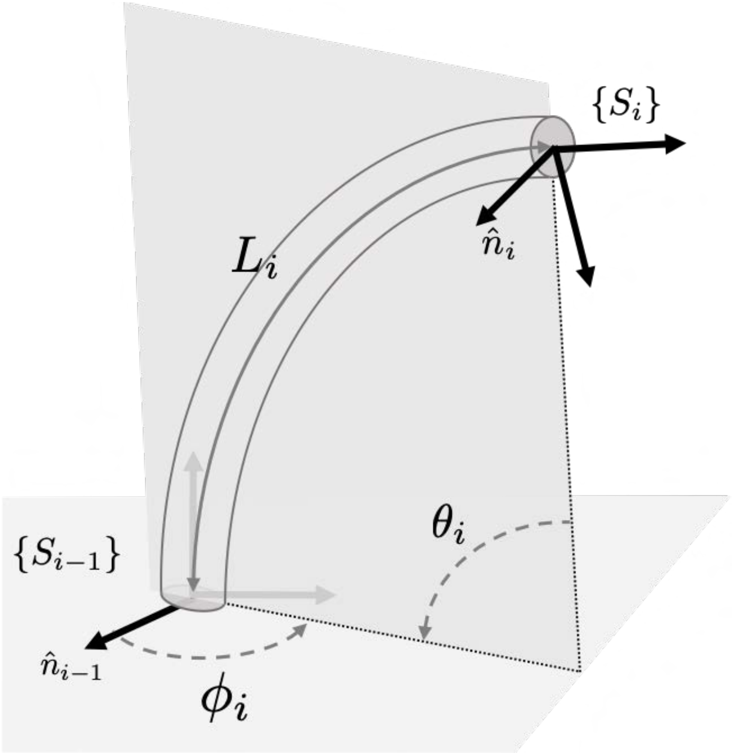
All kinds of pathological behaviors around the straight configuration



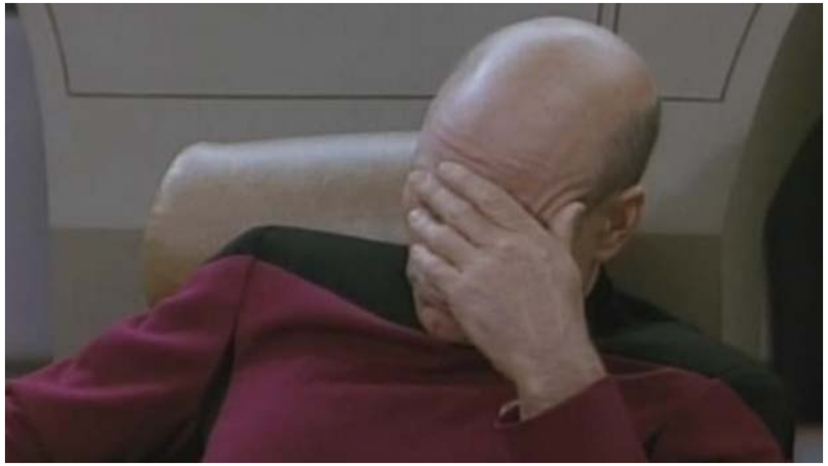
Father of all sins: singular Jacobian

$$
{}^{x}J_{\alpha,i} = \begin{bmatrix}
\frac{s_{\phi_i}\left(1-c_{\theta_i}\right)L_i}{\theta_i} & \frac{c_{\phi_i}L_i\left(1-c_{\theta_i}-\theta_i\, s_{\theta_i}\right)}{\theta_i^2} & \frac{c_{\phi_i}\left(c_{\theta_i}-1\right)}{\theta_i} \\
\frac{c_{\phi_i}\left(c_{\theta_i}-1\right)L_i}{\theta_i} & \frac{s_{\phi_i}L_i\left(1-c_{\theta_i}-\theta_i\, s_{\theta_i}\right)}{\theta_i^2} & \frac{s_{\phi_i}\left(c_{\theta_i}-1\right)}{\theta_i} \\
0 & -\frac{L_i\left(s_{\theta_i}-\theta_i\, c_{\theta_i}\right)}{\theta_i^2} & \frac{s_{\theta_i}}{\theta_i}
\end{bmatrix}
$$

$$
\det({}^{x}J_{\alpha,i}) = -\frac{\left(\cos\left(\theta_i\right)-1\right)^2\left(\delta L_i + L_{0,i}\right)^2}{\theta_i^3}
$$



All kinds of pathological behaviors around the straight configuration



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$$

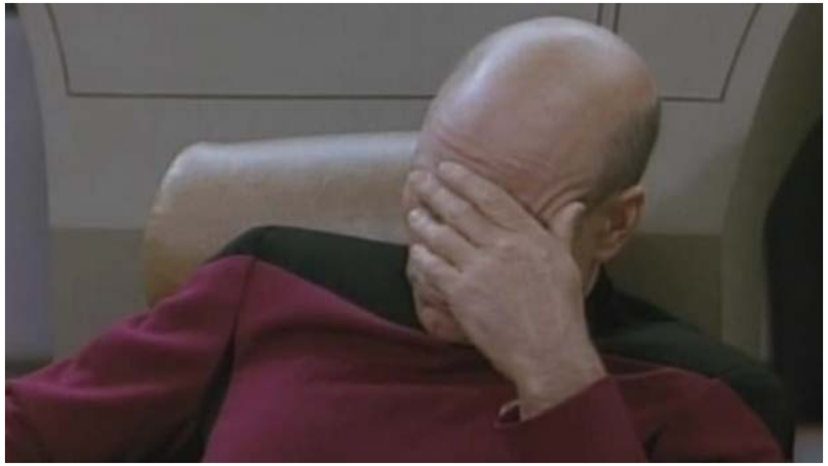
$$
\det({}^{x}J_{\alpha,i}) = -\frac{\left(\cos\left(\theta_i\right)-1\right)^2\left(\delta L_i + L_{0,i}\right)^2}{\theta_i^3}
$$

Singular inertia matrix

$$
\lim_{\theta\to 0} b_{\phi\phi}(\theta,\delta L) \;=\; 0
$$

$$
B_\alpha = \mu\left({}^{x}J_\alpha^{\mathrm{T}}\,{}^{x}J_\alpha\right) = \mu\begin{bmatrix} b_{\phi\phi}(\theta,\delta L) & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{bmatrix}
$$

All kinds of pathological behaviors around the straight configuration



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{}^{x}J_{\alpha,i} = \begin{bmatrix} \frac{s_{\phi_i}\left(1-c_{\theta_i}\right)L_i}{\theta_i} & \frac{c_{\phi_i}L_i\left(1-c_{\theta_i}-\theta_i\, s_{\theta_i}\right)}{\theta_i^2} & \frac{c_{\phi_i}\left(c_{\theta_i}-1\right)}{\theta_i} \\ \frac{c_{\phi_i}\left(c_{\theta_i}-1\right)L_i}{\theta_i} & \frac{s_{\phi_i}L_i\left(1-c_{\theta_i}-\theta_i\, s_{\theta_i}\right)}{\theta_i^2} & \frac{s_{\phi_i}\left(c_{\theta_i}-1\right)}{\theta_i} \\ 0 & -\frac{L_i\left(s_{\theta_i}-\theta_i\, c_{\theta_i}\right)}{\theta_i^2} & \frac{s_{\theta_i}}{\theta_i} \end{bmatrix}
$$

$$
\det({}^{x}J_{\alpha,i}) = -\frac{\left(\cos\left(\theta_i\right)-1\right)^2\left(\delta L_i + L_{0,i}\right)^2}{\theta_i^3}
$$

Singular inertia matrix

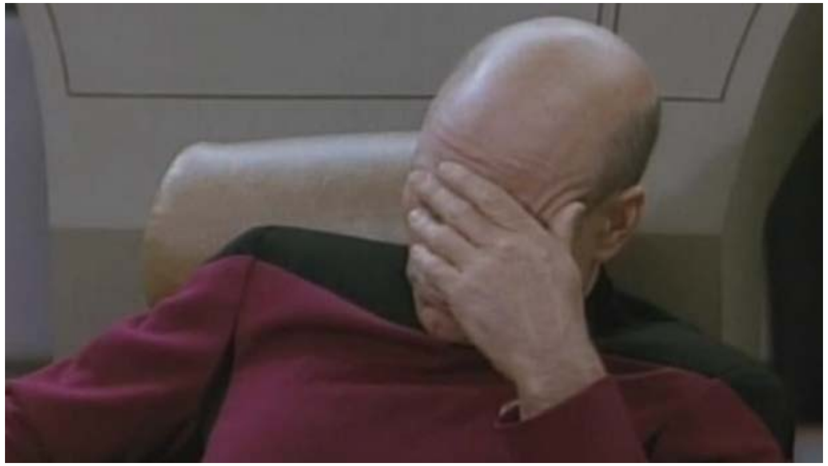
$$
\lim_{\theta\to 0} b_{\phi\phi}(\theta, \delta L) \;=\; 0
$$

$$
B_\alpha = \mu\left({}^{x}J_\alpha^{\mathrm{T}}\,{}^{x}J_\alpha\right) = \mu \begin{bmatrix} b_{\phi\phi}(\theta,\delta L) & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{bmatrix}
$$

Singular input field

$$
A_{\alpha,i}(\phi_i, 0, \delta L_i) = \begin{bmatrix} 0 & 0 & 0 \\ -s_{\phi_i} & c_{\phi_i} & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

All kinds of pathological behaviors around the straight configuration



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$$ \det({}^{x}J_{\alpha,i}) = -\frac{\left(\cos\left(\theta_i\right)-1\right)^2\left(\delta L_i + L_{0,i}\right)^2}{\theta_i^3} $$

Singular inertia matrix

$$ \lim_{\theta\to 0} b_{\phi\phi}(\theta,\delta L) = 0 $$

$$ B_\alpha = \mu\left({}^{x}J_\alpha^{\mathrm{T}}\,{}^{x}J_\alpha\right) = \mu\begin{bmatrix} b_{\phi\phi}(\theta,\delta L) & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{bmatrix} $$

Singular and non linear impedance

$$ K_{\alpha,i} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \kappa_{\theta,i} & 0 \\ 0 & 0 & \kappa_{\delta L,i} \end{bmatrix} \qquad D_{\alpha,i}(\theta_i) = \begin{bmatrix} \beta_{\theta,i}\theta_i^2 & 0 & 0 \\ 0 & \beta_{\theta,i} & 0 \\ 0 & 0 & \beta_{\delta L,i} \end{bmatrix} $$

Singular input field

$$ A_{\alpha,i}(\phi_i, 0, \delta L_i) = \begin{bmatrix} 0 & 0 & 0 \\ -s_{\phi_i} & c_{\phi_i} & 0 \\ 0 & 0 & 1 \end{bmatrix} $$

Model Based Feedback Controller

$$\tau_{\mathrm{A}} = A^{-1}(q)\left(G_{\mathrm{G}}(q) + C(q,\dot{q})\dot{\bar{q}} + B(q)\ddot{\bar{q}} + K\bar{q} + K_{\mathrm{P}}(\bar{q} - q) + D(q)\dot{\bar{q}} + K_{\mathrm{D}}(\dot{\bar{q}} - \dot{q})\right)$$

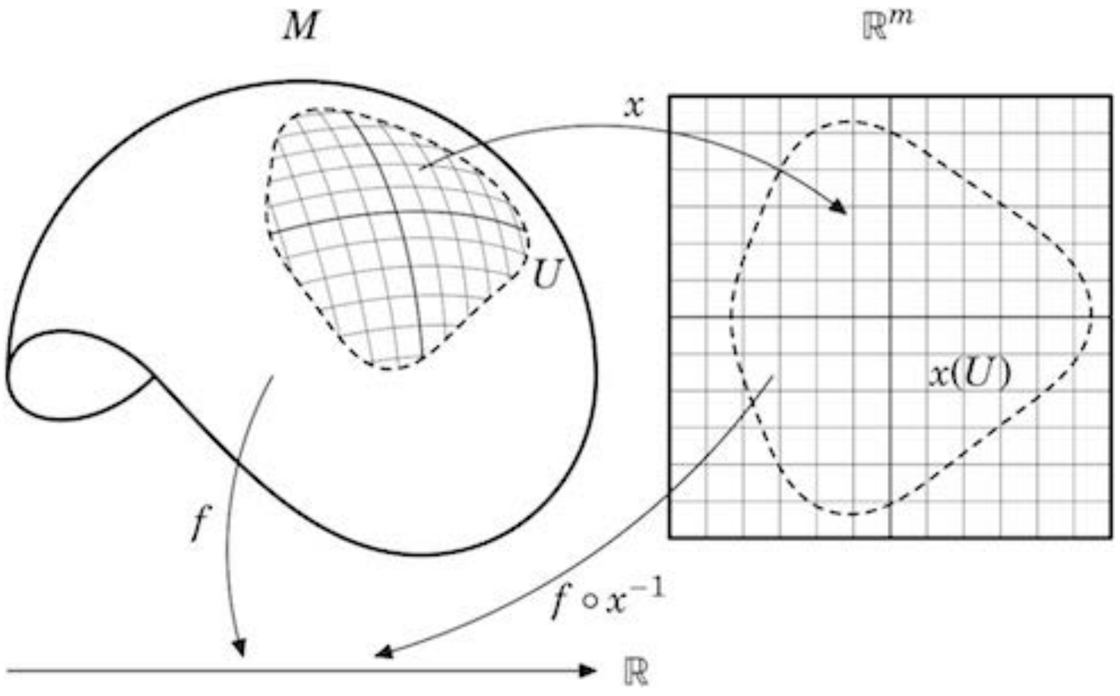
Della Santina, Cosimo, Antonio Bicchi, and Daniela Rus. "On an improved state parametrization for soft robots with piecewise constant curvature and its use in model based control." *IEEE Robotics and Automation Letters* 5.2 (2020): 1001-1008.

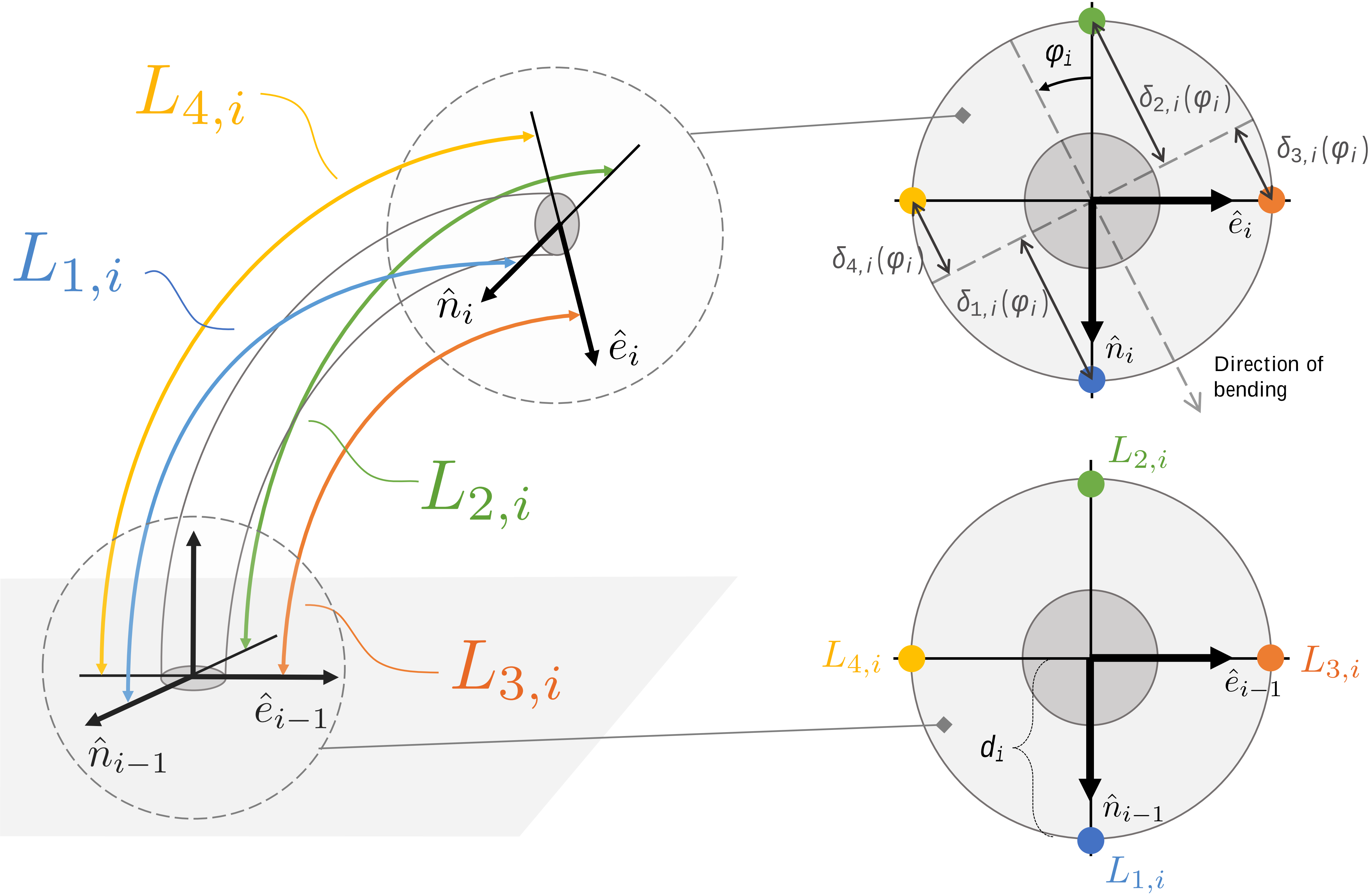
Straight Configuration


Side View
1x

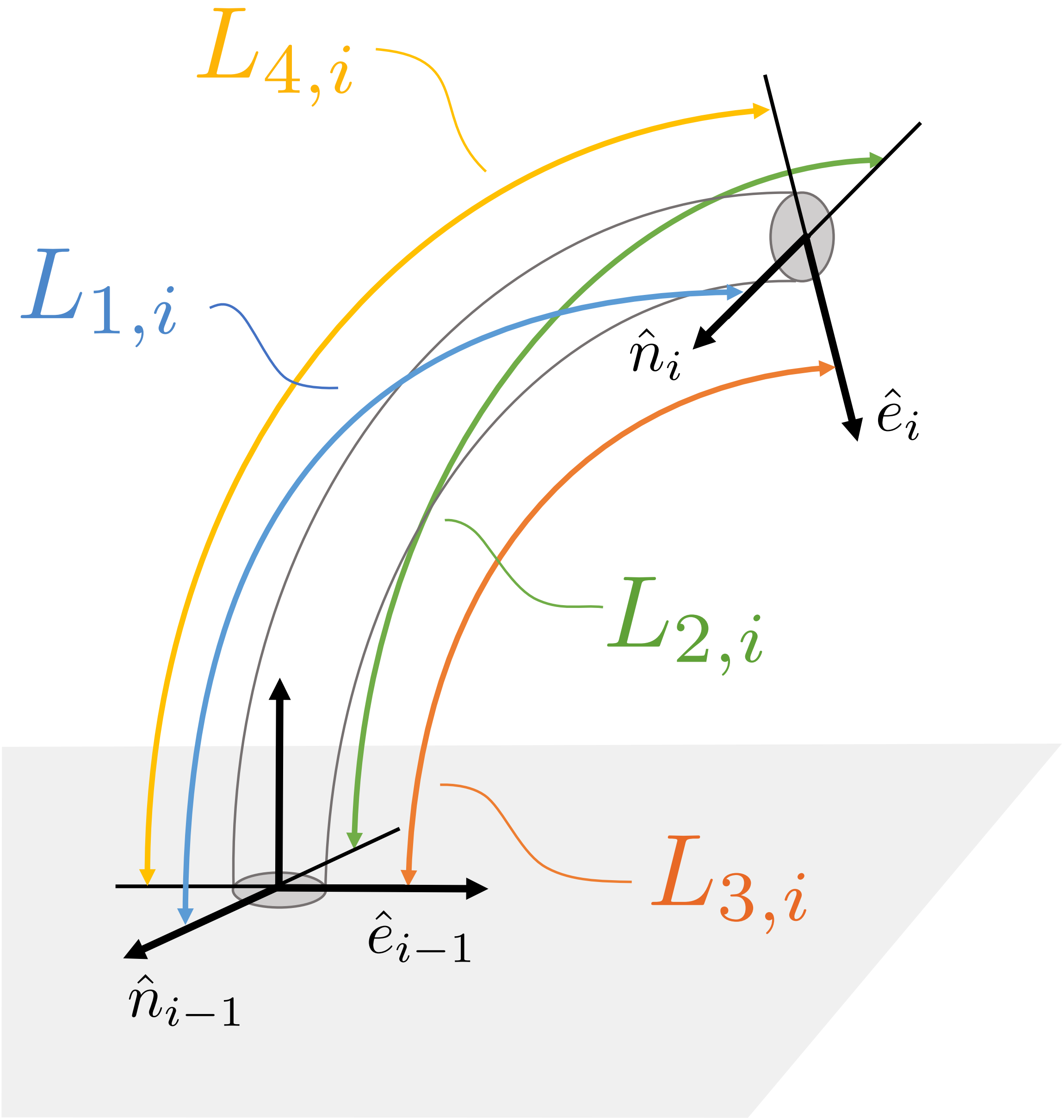

YOU SHALL NOT PASS!

The configuration manifold is a cup, not a sphere





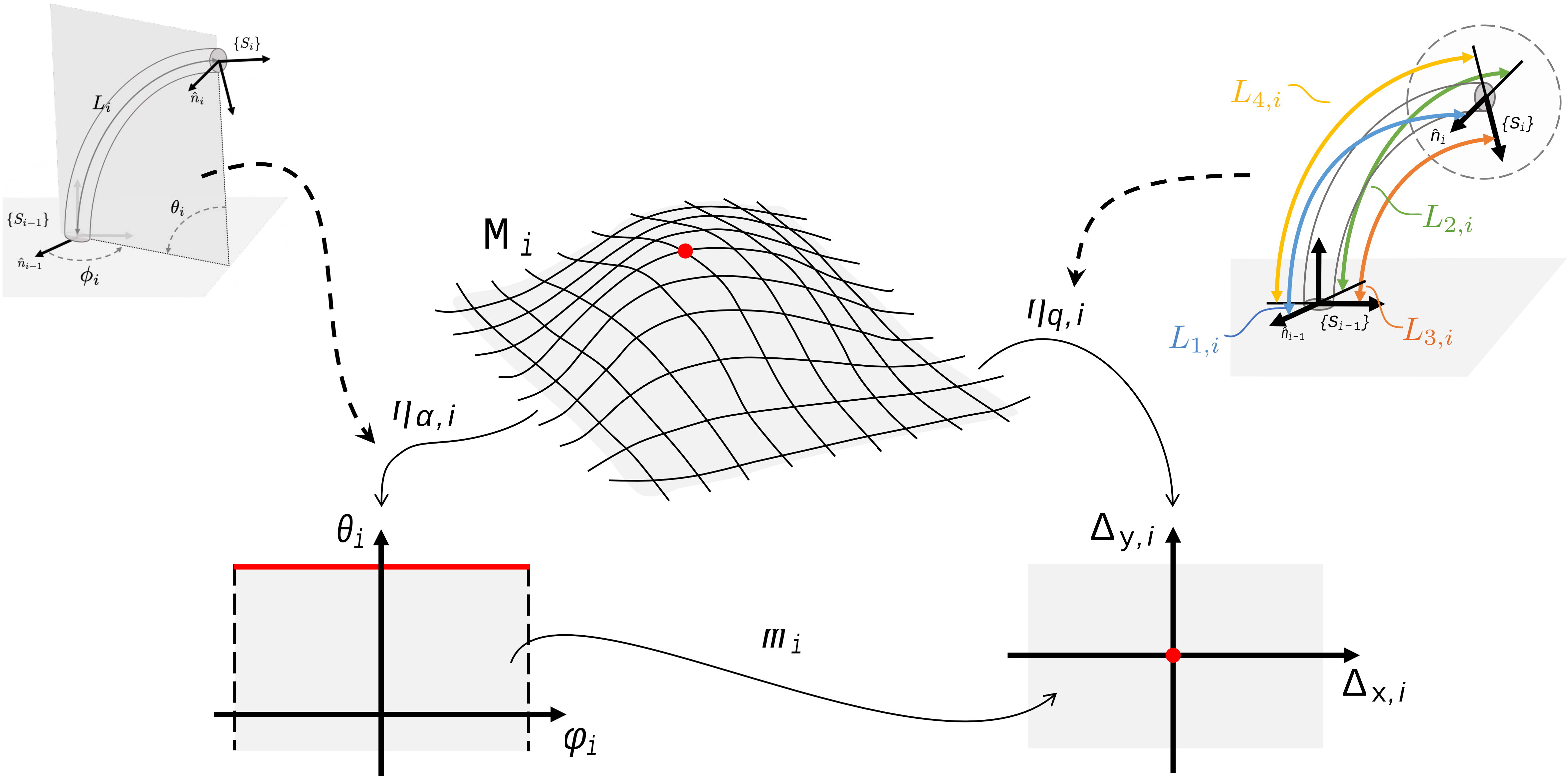
$L_{4,i}$
$L_{1,i}$
$\hat{n}_i$
$\hat{e}_i$
$L_{2,i}$
$L_{3,i}$
$\hat{e}_{i-1}$
$\hat{n}_{i-1}$
φ_i
$\delta_{2,i}(\varphi_i)$
$\delta_{3,i}(\varphi_i)$
$\delta_{4,i}(\varphi_i)$
$\delta_{1,i}(\varphi_i)$
$\hat{e}_i$
$\hat{n}_i$
Direction of bending
$L_{2,i}$
$L_{4,i}$
$L_{3,i}$
$\hat{e}_{i-1}$
d_i
$\hat{n}_{i-1}$
$L_{1,i}$



$$\Delta_{x,i} = \frac{L_{2,i} - L_{1,i}}{2}$$

$$\Delta_{y,i} = \frac{L_{4,i} - L_{3,i}}{2}$$

Della Santina, Cosimo, Antonio Bicchi, and Daniela Rus. "On an improved state parametrization for soft robots with piecewise constant curvature and its use in model based control." *IEEE Robotics and Automation Letters* 5.2 (2020): 1001-1008.



Della Santina, Cosimo, Antonio Bicchi, and Daniela Rus. "On an improved state parametrization for soft robots with piecewise constant curvature and its use in model based control." *IEEE Robotics and Automation Letters* 5.2 (2020): 1001-1008.

Full rank Jacobian!!!

$$\left(\frac{\delta L_i + L_{0,i}}{2}\right)^2$$

$$\det({}^{x}J_q) = \frac{\left(\cos\left(\sqrt{\frac{\Delta_{\mathrm{x},i}{}^2 + \Delta_{\mathrm{y},i}{}^2}{d_i}}\right) - 1\right)^2 (\delta L_i + L_{0,i})^2}{\left(\Delta_{\mathrm{x},i}{}^2 + \Delta_{\mathrm{y},i}{}^2 / d_i^2\right)^2}$$

Full rank inertia matrix

$$\lim_{\substack{\Delta_{\mathrm{x}} \to 0 \\ \Delta_{\mathrm{y}} \to 0}} B(\Delta_{\mathrm{x}}, \Delta_{\mathrm{y}}, \delta L) = \mu \begin{bmatrix} \frac{(\delta L + L_0)^2}{4} & 0 & 0 \\ 0 & \frac{(\delta L + L_0)^2}{4} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

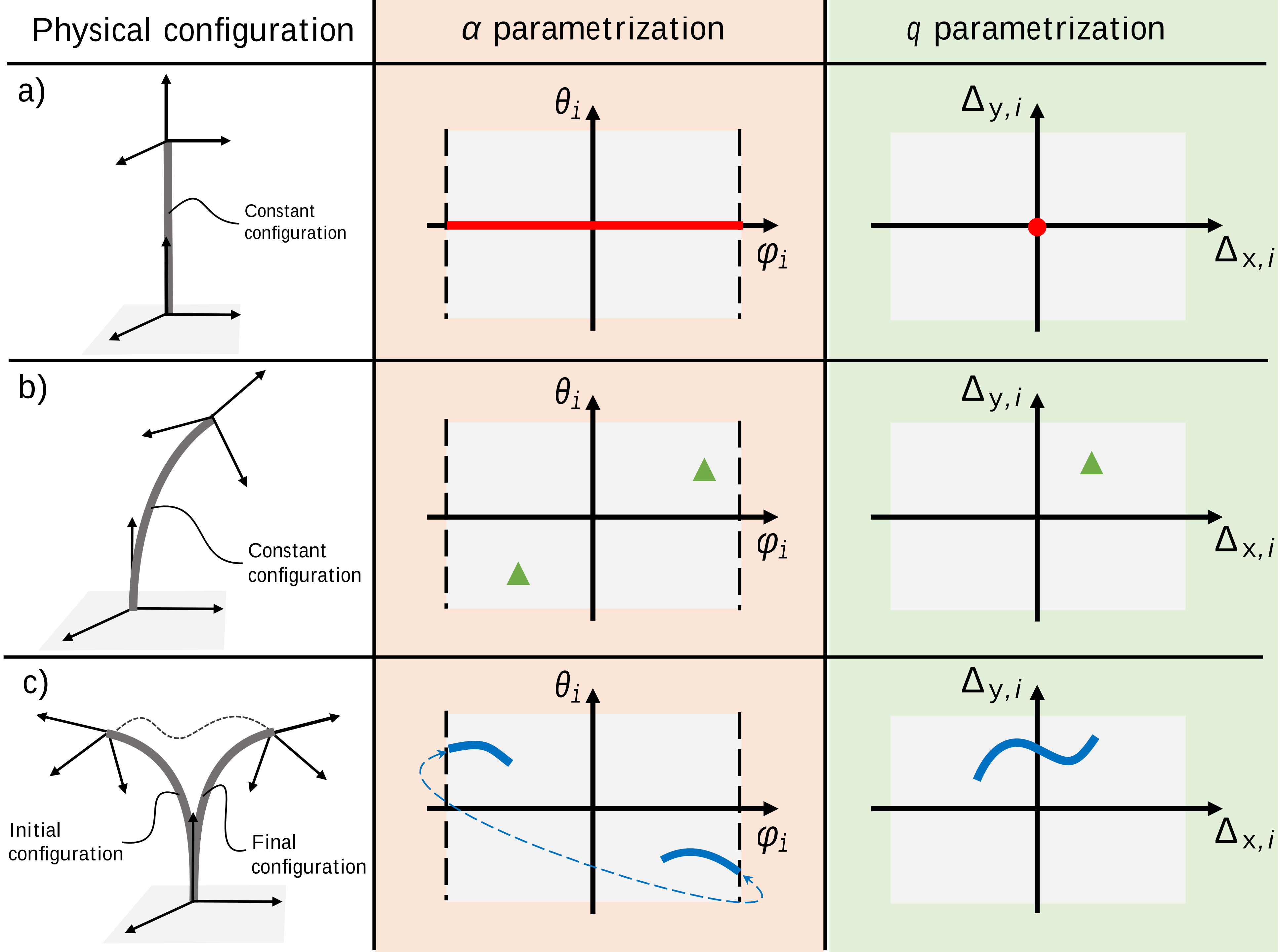
Full rank and linear impedance

$${}^{\alpha}J_{q,i}^{\mathrm{T}}(q) K_{\alpha,i}\, m_i(q_i) = \begin{bmatrix} \kappa_{\theta,i} & 0 & 0 \\ 0 & \kappa_{\theta,i} & 0 \\ 0 & 0 & \kappa_{\delta L,i} \end{bmatrix} \begin{bmatrix} \Delta_{\mathrm{x},i} \\ \Delta_{\mathrm{y},i} \\ \delta L_i \end{bmatrix} \doteq K_i q$$

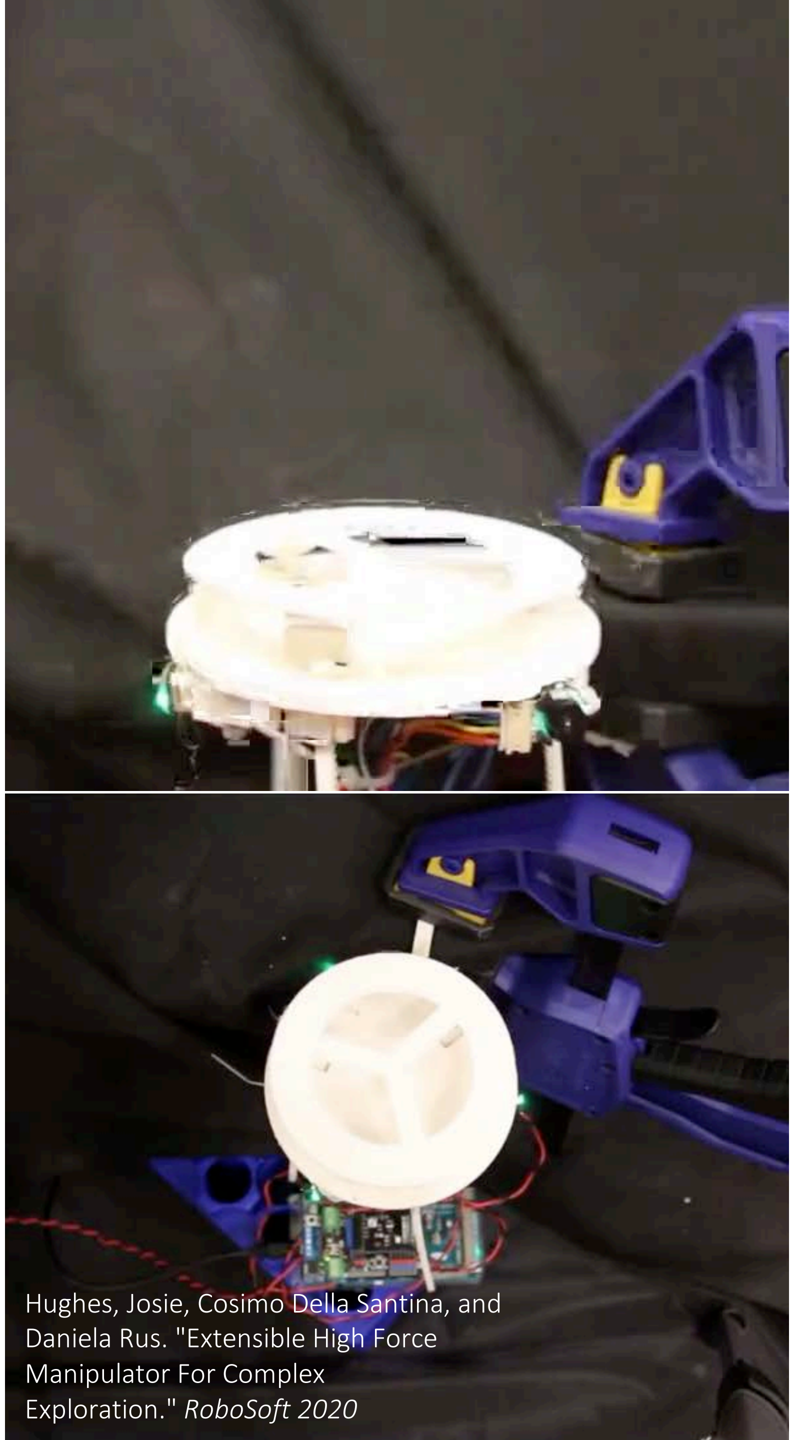
$$D_i \doteq {}^{\alpha}J_{q,i}^{\mathrm{T}}(q) D_{\alpha,i}(m_i(q_i))\, {}^{\alpha}J_{q,i}(q_i) = \begin{bmatrix} \beta_{\theta,i} & 0 & 0 \\ 0 & \beta_{\theta,i} & 0 \\ 0 & 0 & \beta_{\delta L,i} \end{bmatrix},$$

Full rank input field

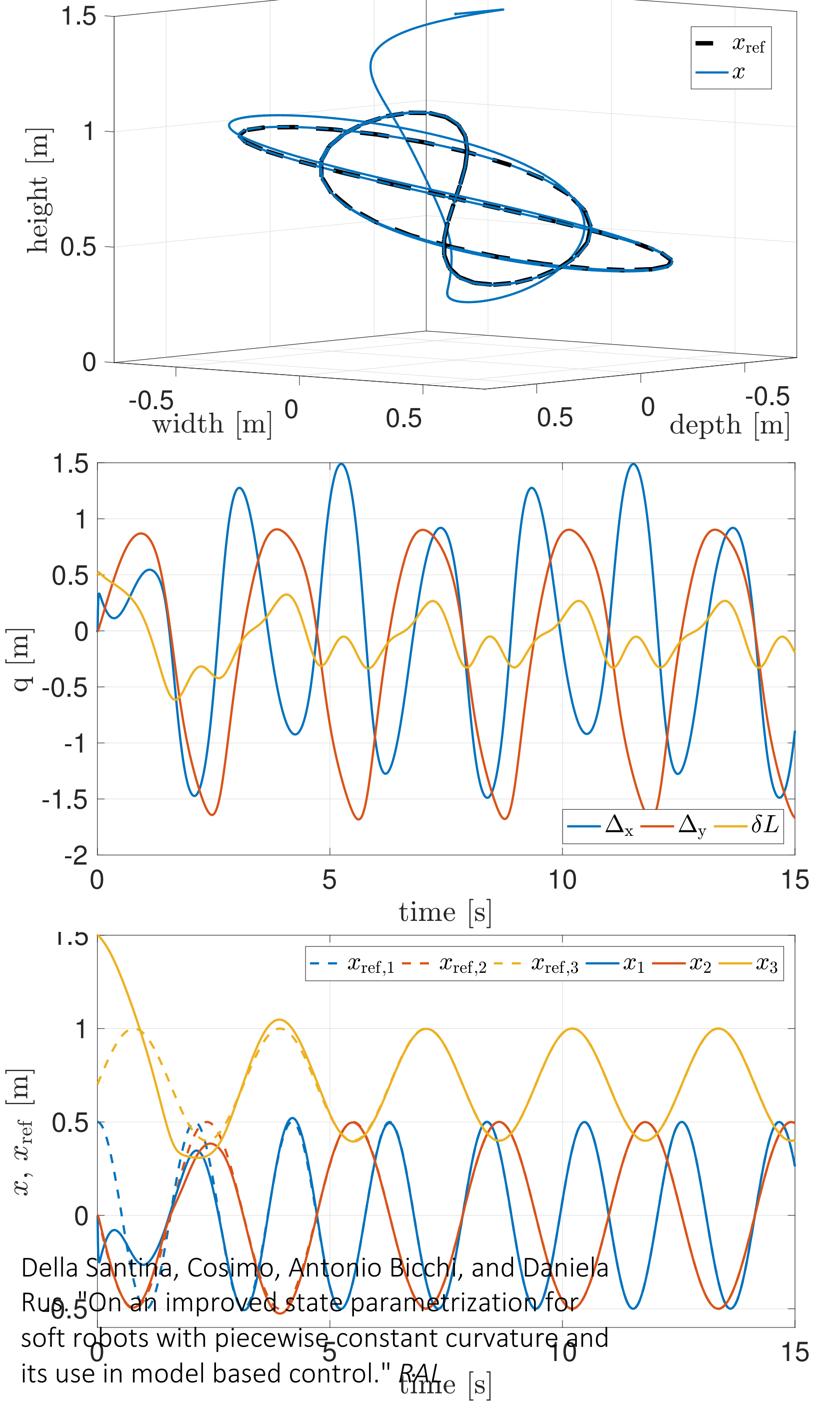
$$A(q) = \begin{bmatrix} \frac{\Delta_{\mathrm{x},i}\, \Delta_{\mathrm{y},i}\, D_i}{\Delta_i^3} & \frac{-\Delta_{\mathrm{x},i}^2\, \Delta_i - \Delta_{\mathrm{y},i}^2\, \sin(\Delta_i)}{\Delta_i^3} & \frac{\Delta_{\mathrm{x},i}\, D_i\, L_i}{\Delta_i^3} \\ \frac{\Delta_{\mathrm{y},i}^2\, \Delta_i + \Delta_{\mathrm{x},i}^2\, \sin(\Delta_i)}{\Delta_i^3} & \frac{-\Delta_{\mathrm{x},i}\, \Delta_{\mathrm{y},i}\, D_i}{\Delta_i^3} & \frac{\Delta_{\mathrm{y},i}\, D_i\, L_i}{\Delta_i^3} \\ 0 & 0 & \frac{\sin(\Delta_i)}{\Delta_i} \end{bmatrix}$$


Physical configuration
α parametrization
q parametrization
a)
Constant configuration
θ_i
φ_i
Δ_y,i
Δ_x,i
b)
Constant configuration
θ_i
φ_i
Δ_y,i
Δ_x,i
c)
Initial configuration
Final configuration
θ_i
φ_i
Δ_y,i
Δ_x,i

Li S., […], **Della Santina C.**, et al. "Dynamic control of soft robots with internal constraints in the presence of obstacles." *Submitted to SoRo*



Hughes, Josie, Cosimo Della Santina, and Daniela Rus. "Extensible High Force Manipulator For Complex Exploration." *RoboSoft 2020*



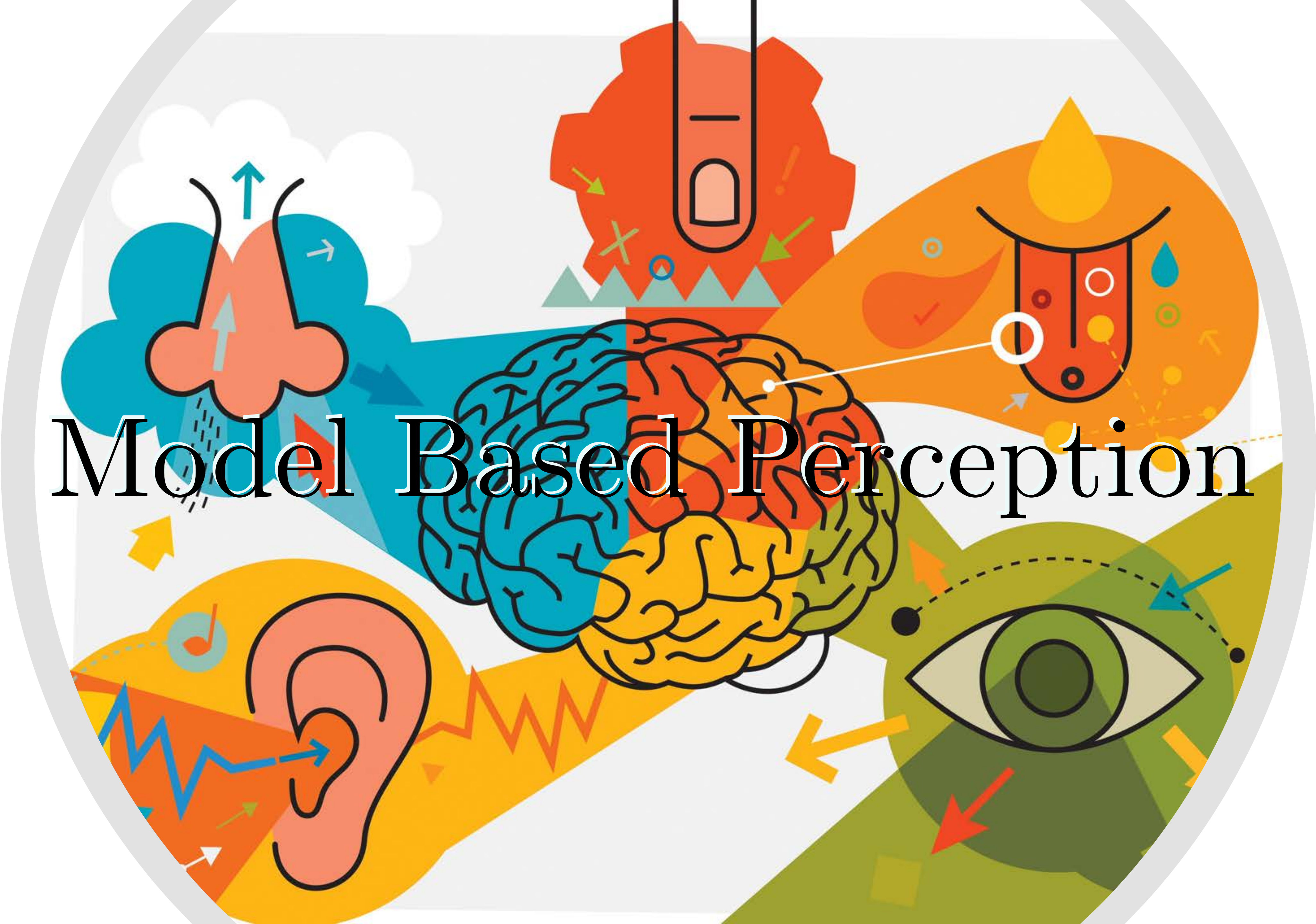
Della Santina, Cosimo, Antonio Bicchi, and Daniela Rus. "On an improved state parametrization for soft robots with piecewise constant curvature and its use in model based control." *RAL*

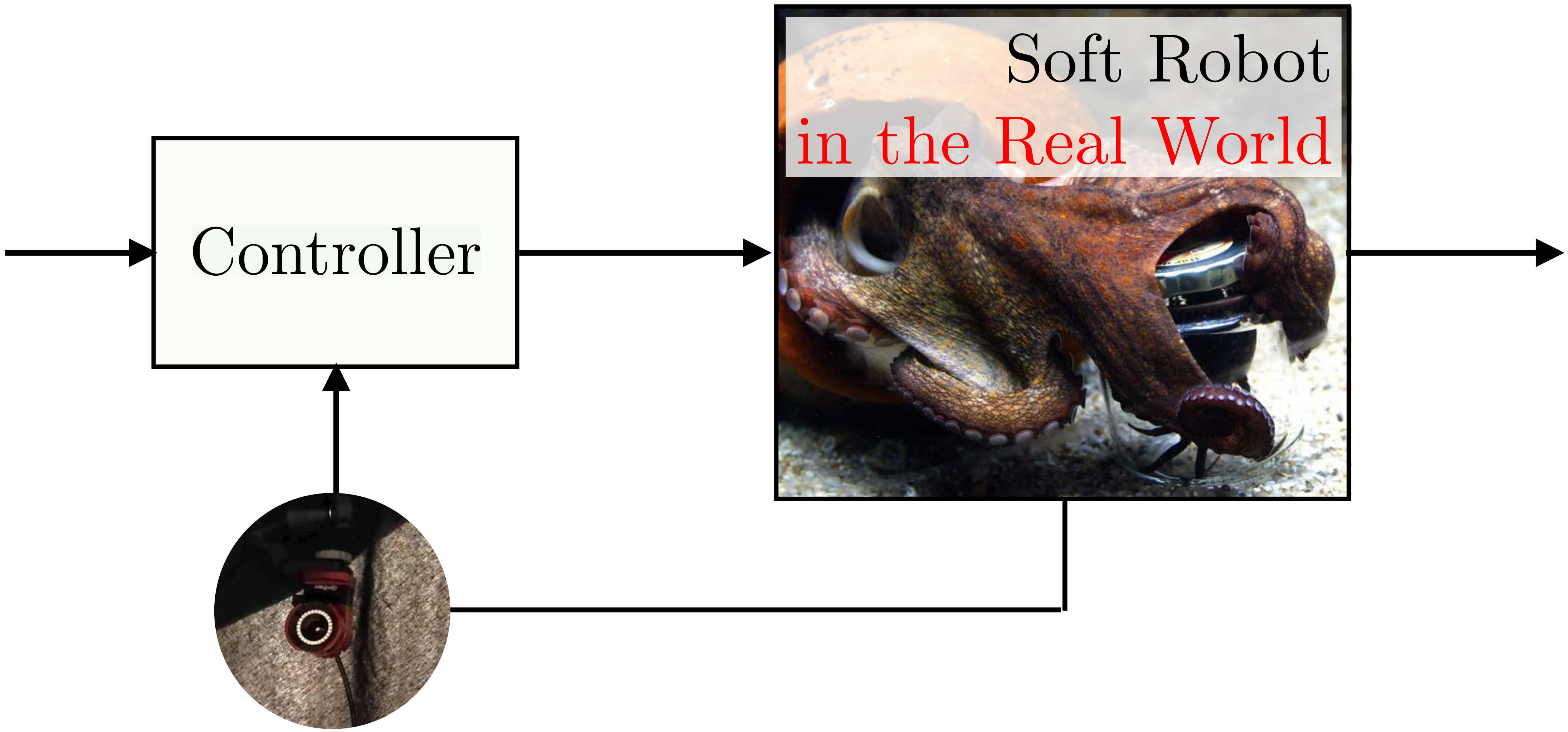
...and large scale soft locomotion!

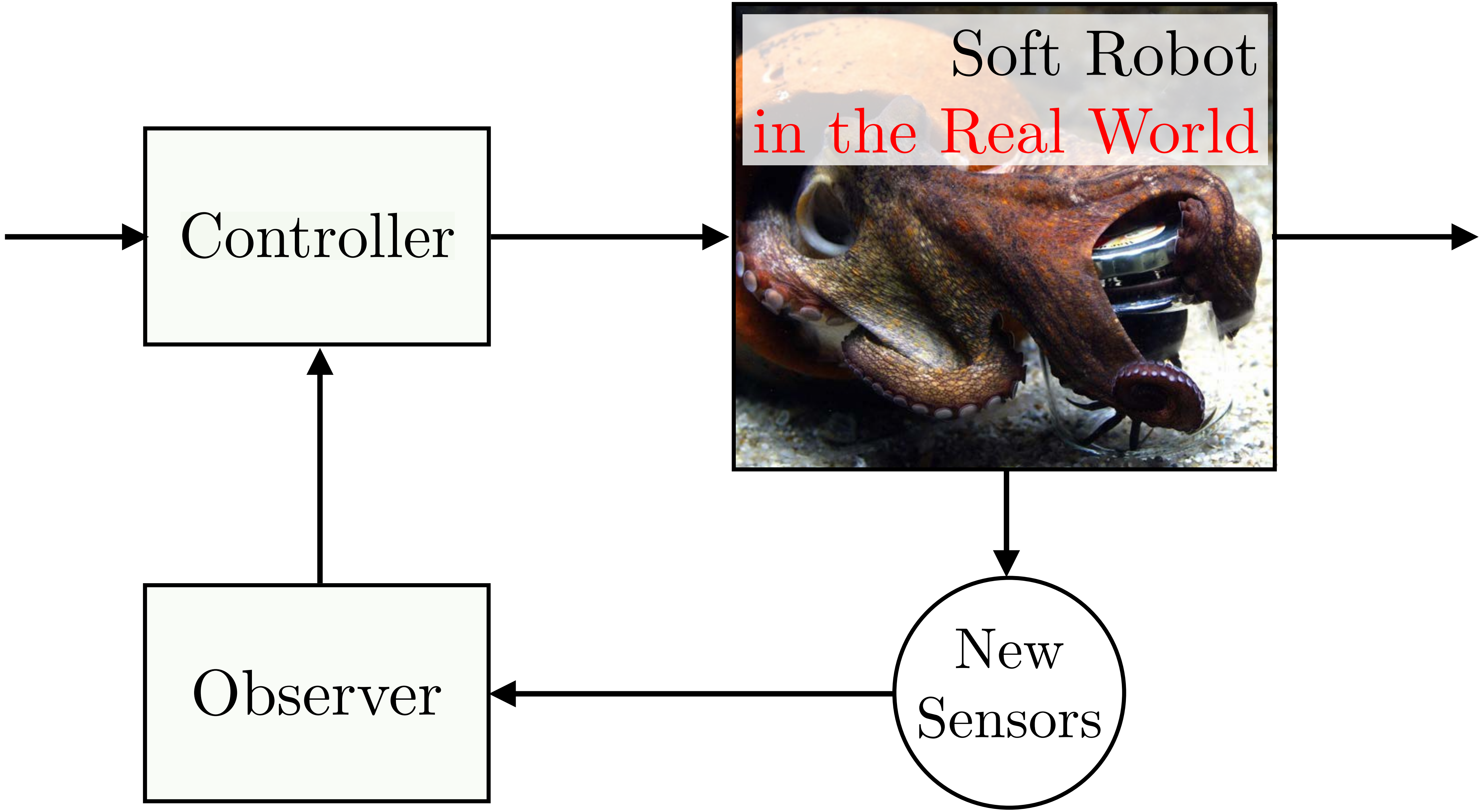
20x

Li S., […], **Della Santina C.**, et al. "Dynamic control of soft robots with internal constraints in the presence of obstacles." *Submitted to SoRo*

Model Based Perception



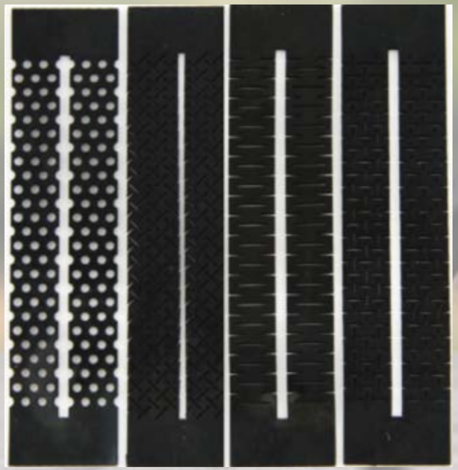

Controller
Soft Robot
in the Real World


Controller
Soft Robot
in the Real World
Observer
New
Sensors

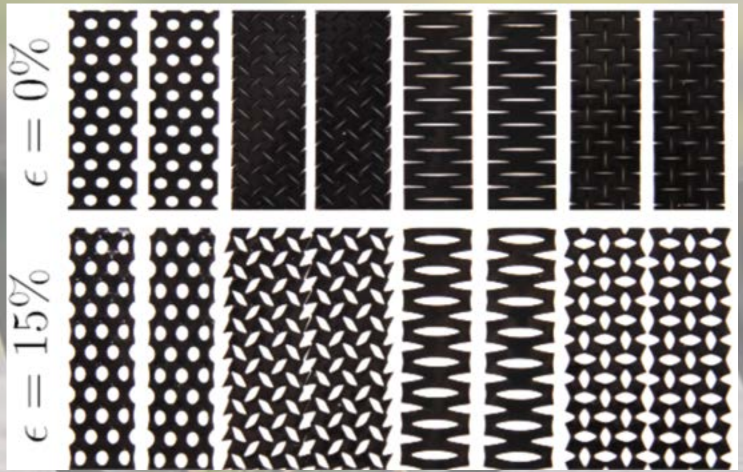
Kirigami enables a simple, rapid approach for sensorizing soft robots

Sensor design and fabrication

Circle 45/-45 1D 0/90

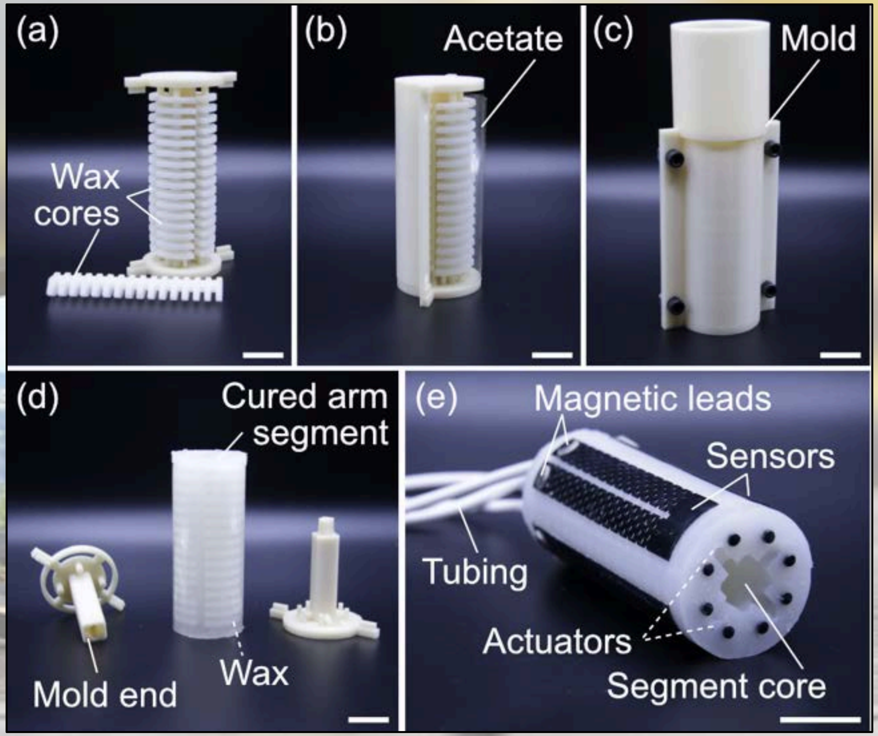


Circle 45/-45 1D 0/90



Electrically conductive silicone (Silex):
Shore A hardness 65±5, volume resistivity ~5 Ω-cm

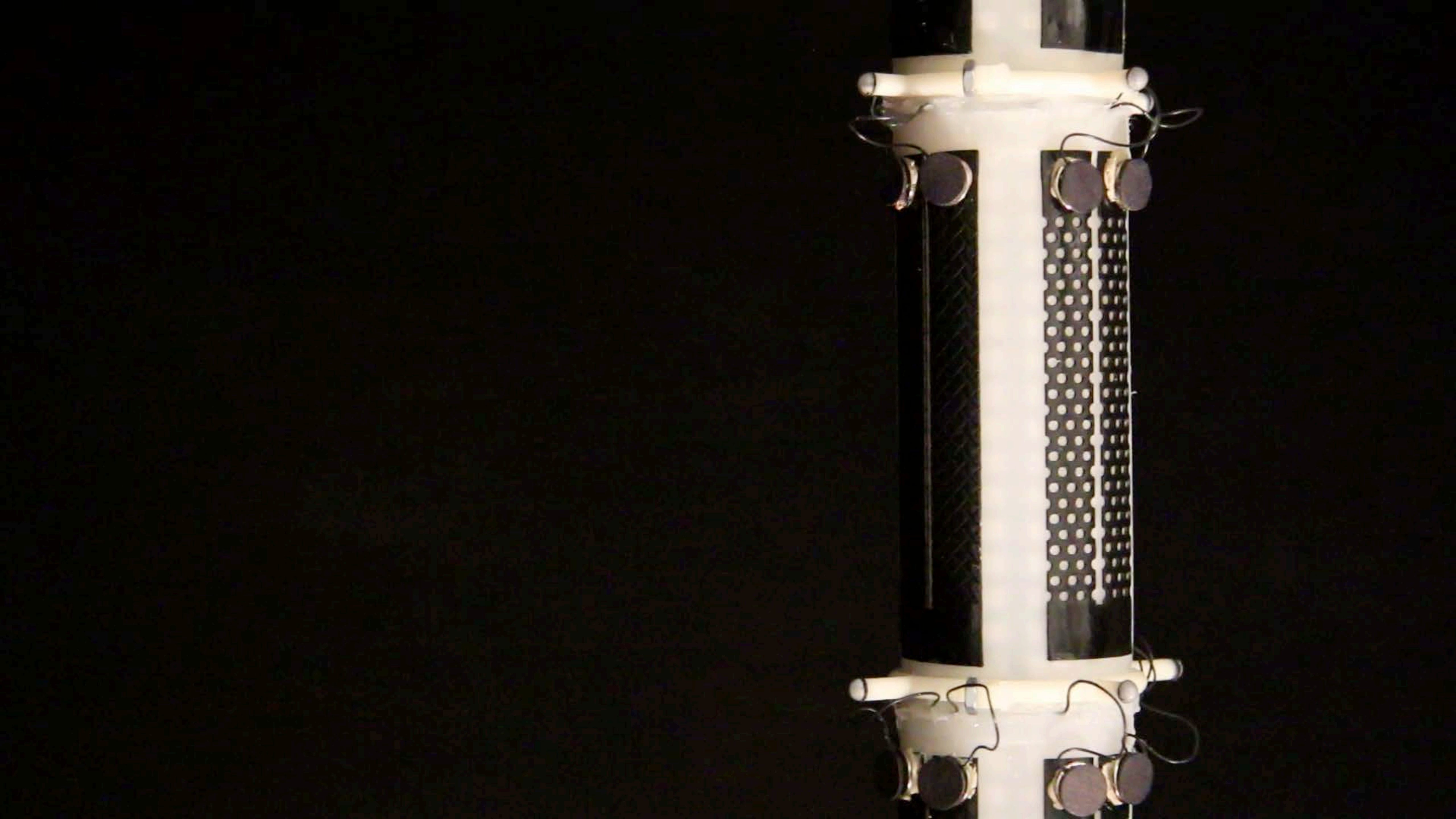
Arm fabrication via lost-wax casting

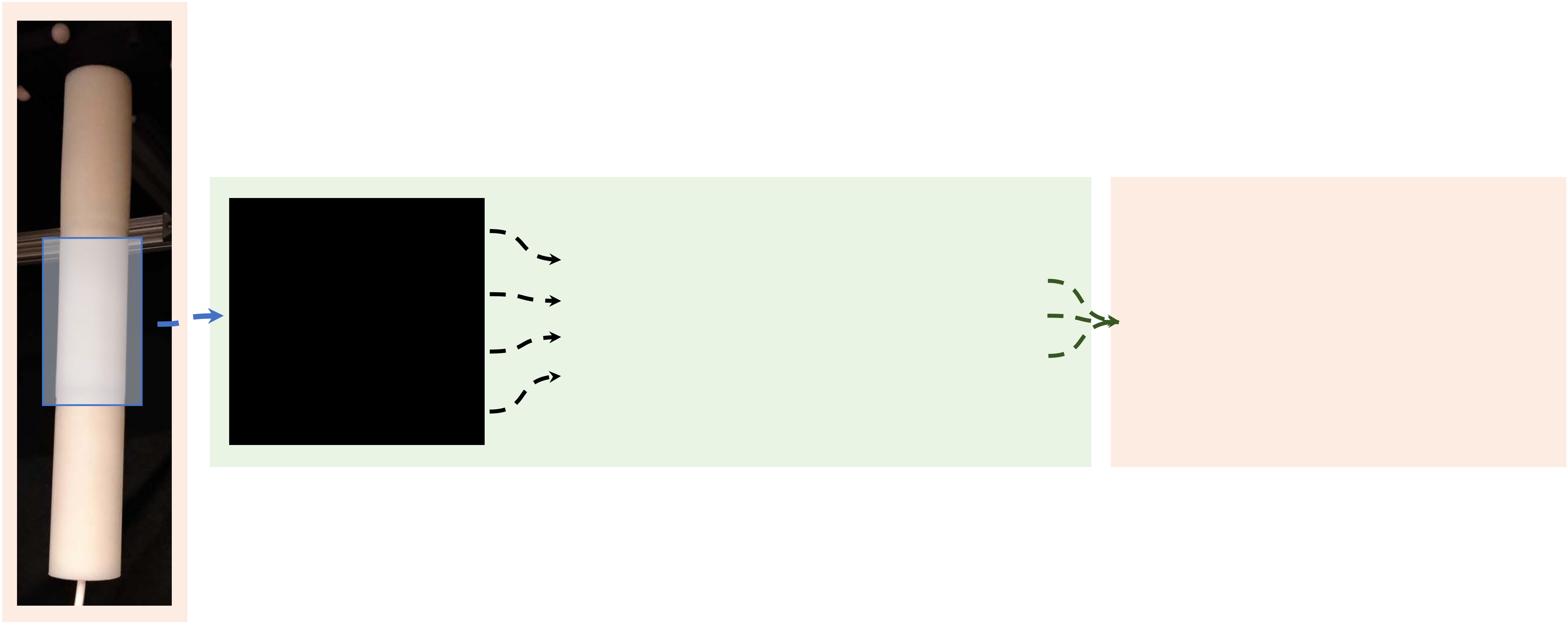


(Hypothesis: Sensor response dependent on cut pattern)

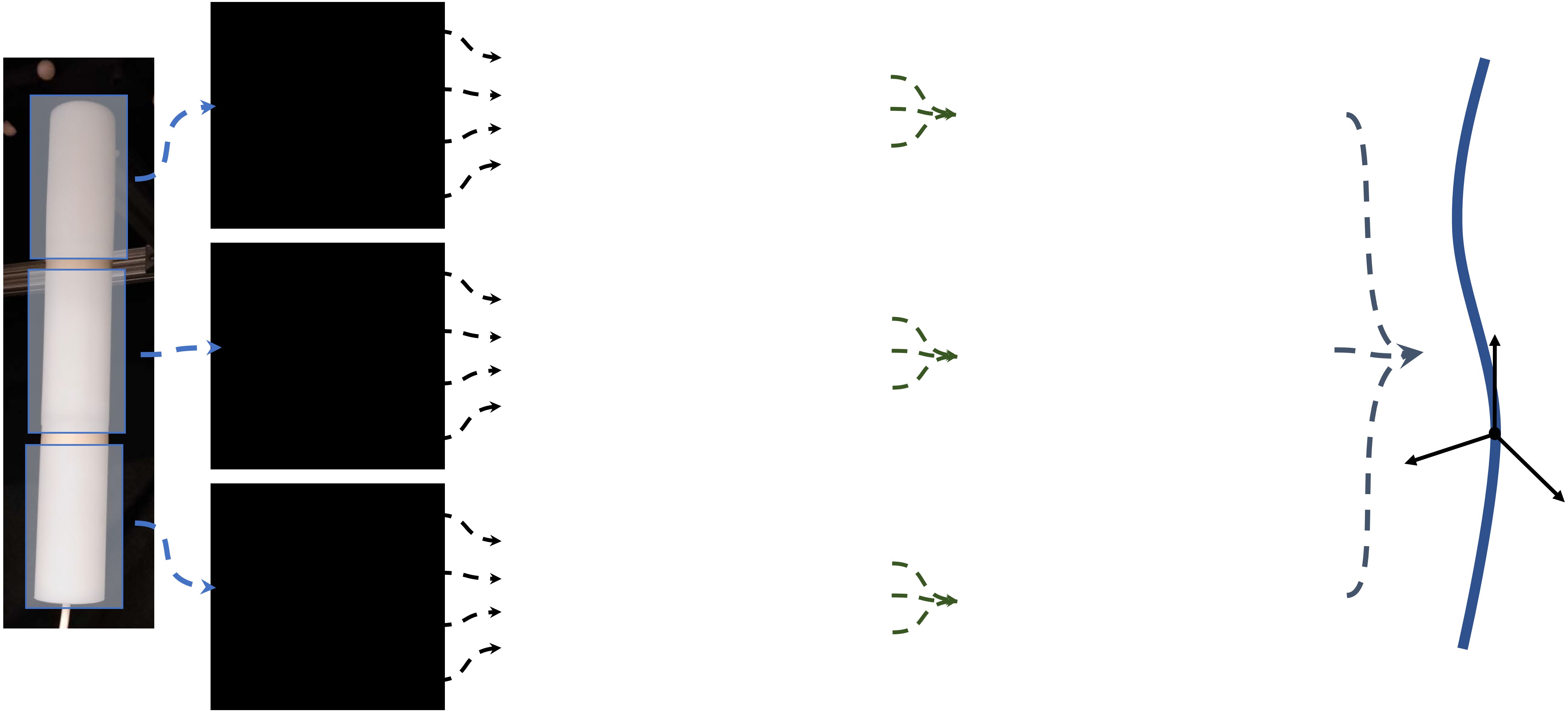
No materials handling or formulation required: sensors are laser cut and done

RL Truby*, C. Della Santina*, D. Rus. *IEEE Robotics and Automation Letters*, 2020. Accepted for IEEE ICRA 2020.





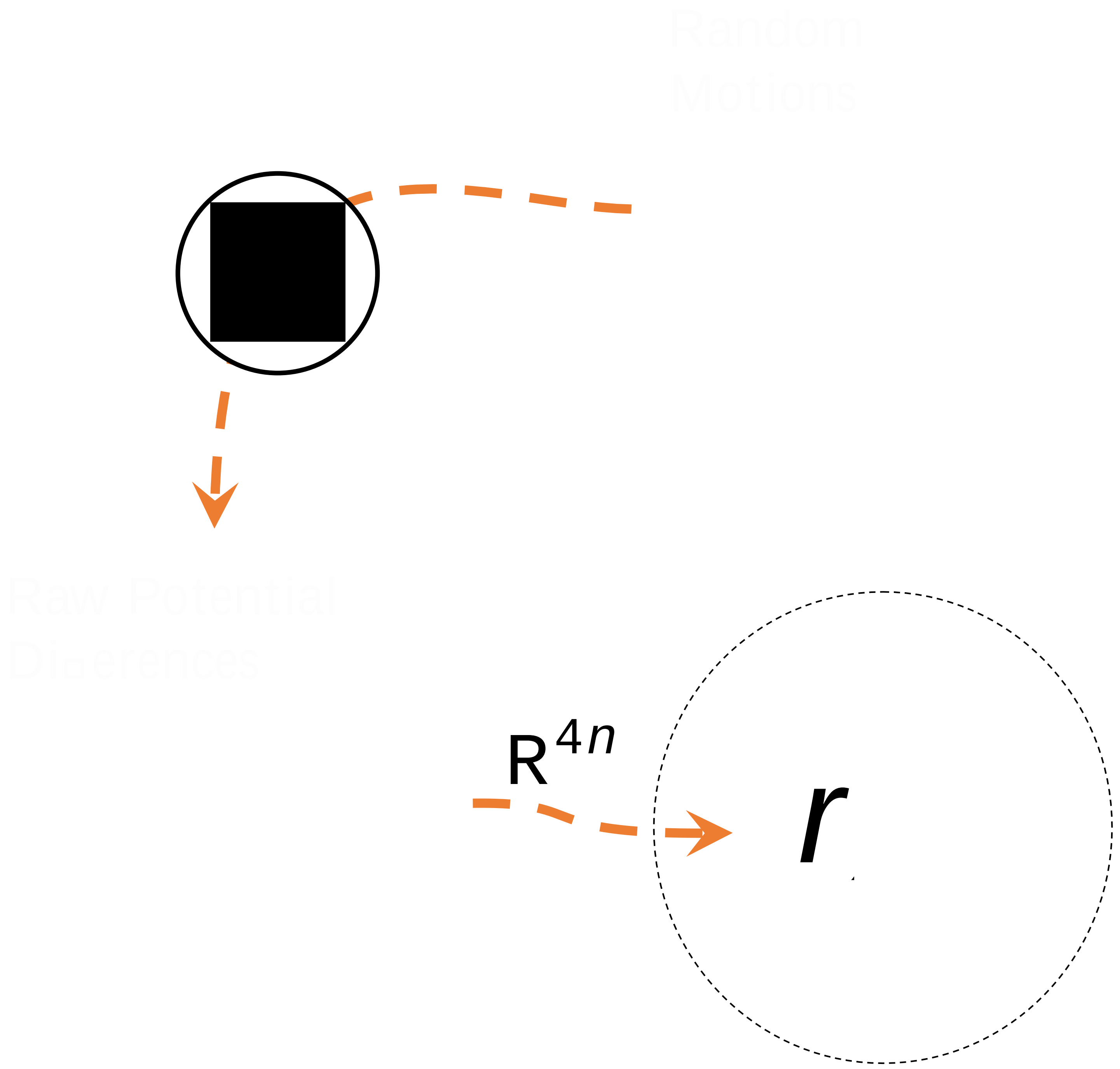
RL Truby*, C. Della Santina*, D. Rus. *IEEE Robotics and Automation Letters*, 2020. Accepted for IEEE ICRA 2020.

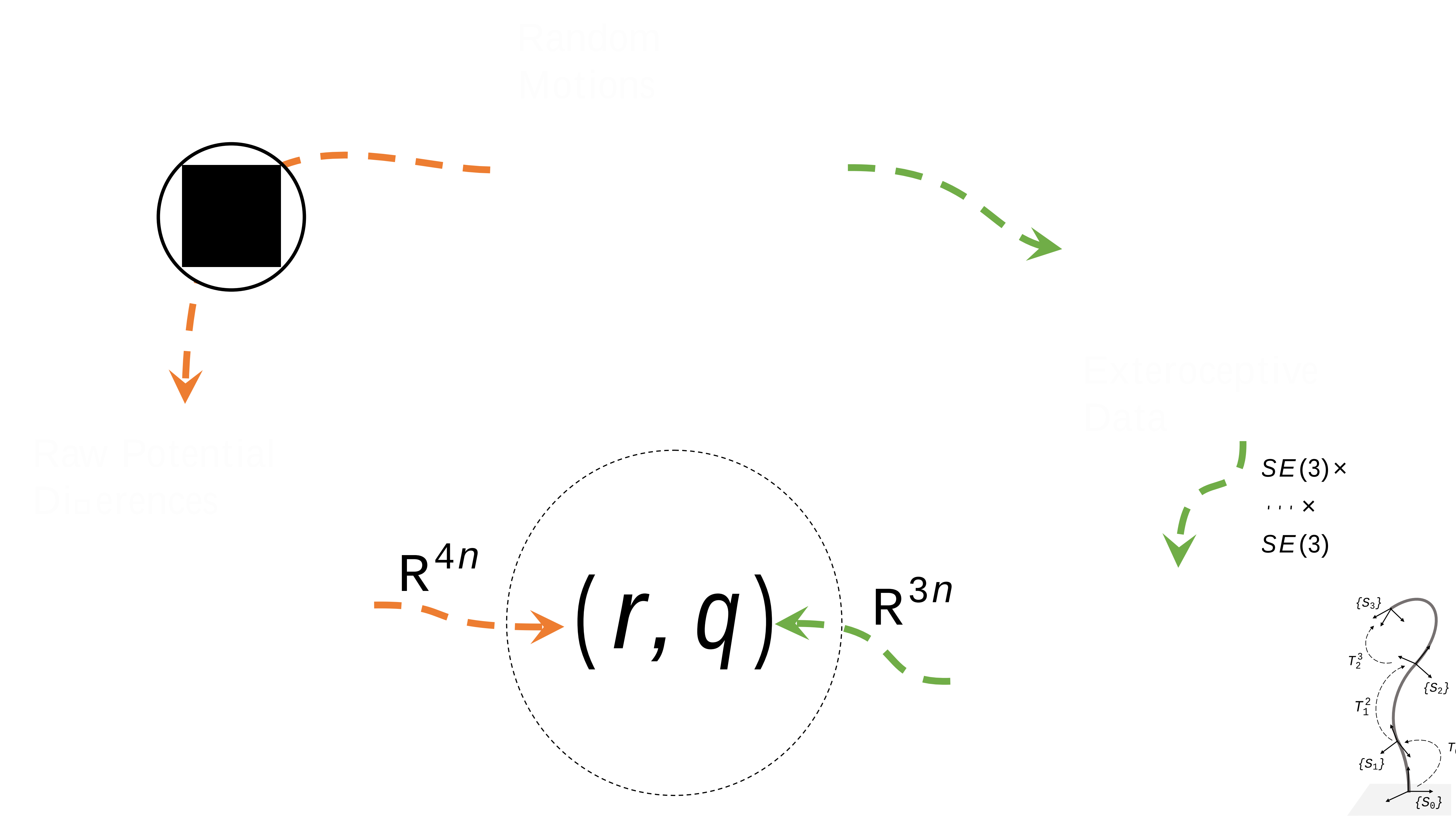


RL Truby*, C. Della Santina*, D. Rus. *IEEE Robotics and Automation Letters*, 2020. Accepted for IEEE ICRA 2020.

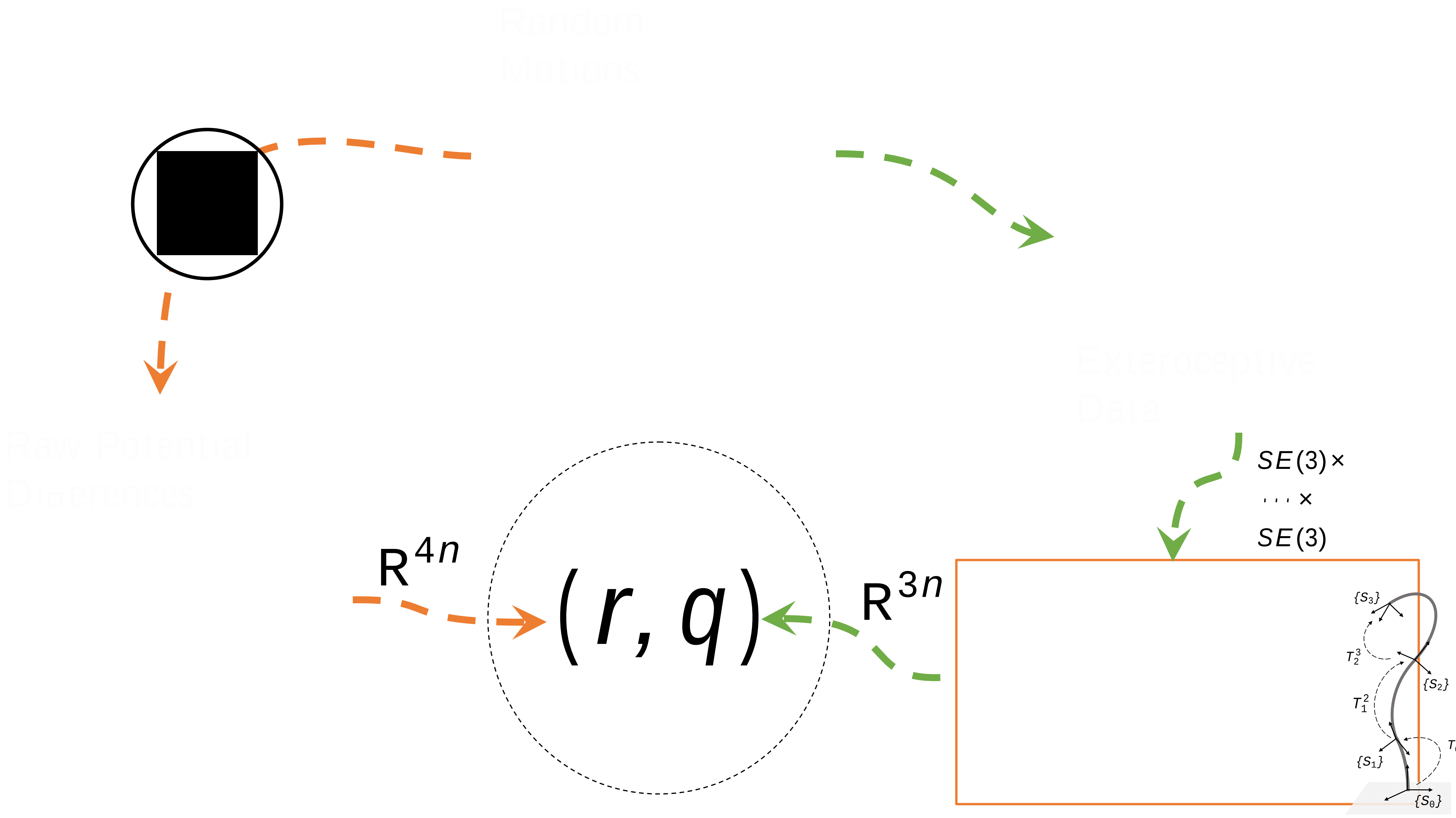

THIS IS YOUR MACHINE LEARNING SYSTEM?
YUP! YOU POUR THE DATA INTO THIS BIG PILE OF LINEAR ALGEBRA, THEN COLLECT THE ANSWERS ON THE OTHER SIDE.
WHAT IF THE ANSWERS ARE WRONG?
JUST STIR THE PILE UNTIL THEY START LOOKING RIGHT.
DATA
ANSWERS

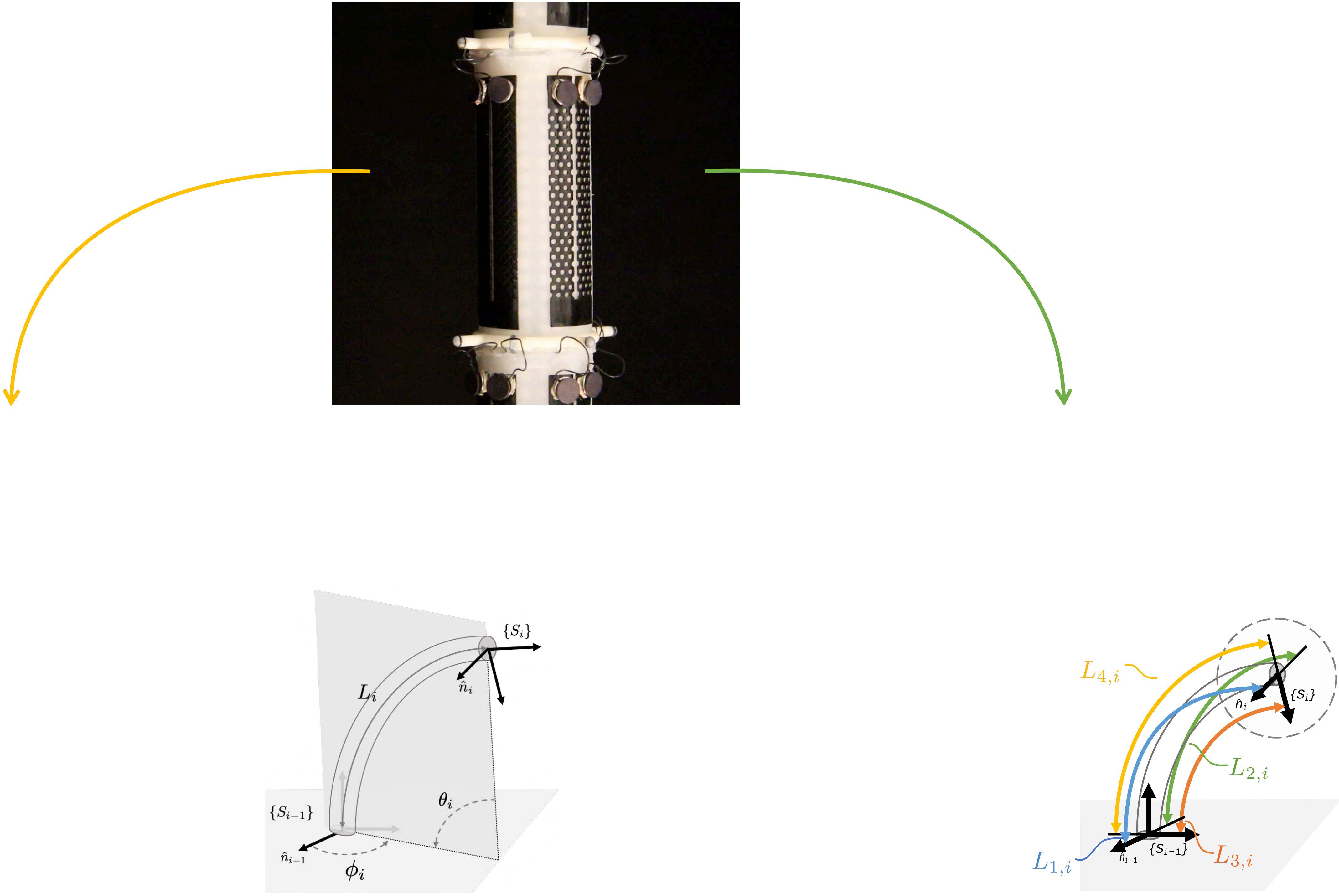
Dataset


Random
Motions
Raw Potential
R4n
r



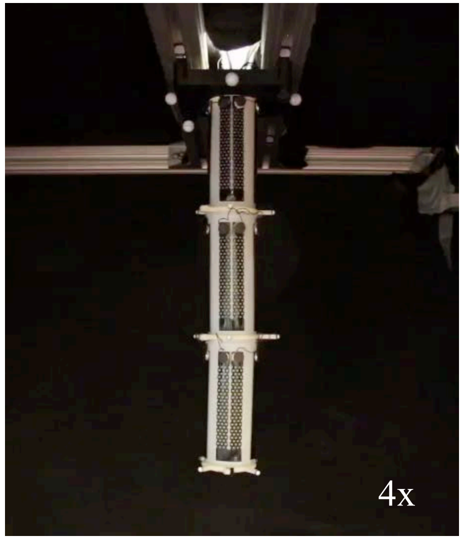
SE(3)×
···×
SE(3)
R^{4n}
(r, q)
R^{3n}
{S_3}
T_2^3
{S_2}
T_1^2
{S_1}
{S_0}

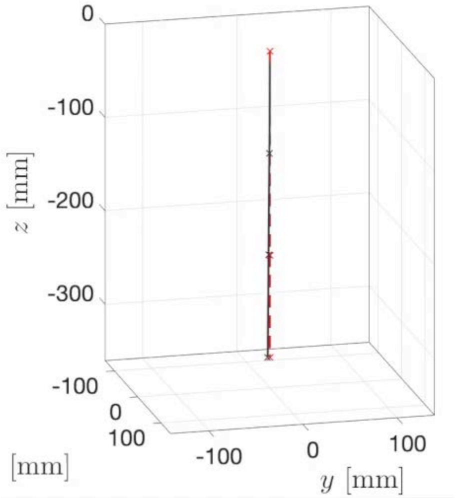

Random Motions
Raw Potential Differences
Exteroceptive Data
$SE(3) \times \cdots \times SE(3)$
$\mathbb{R}^{4n}$
(r, q)
$\mathbb{R}^{3n}$
$\{S_3\}$
T_2^3
$\{S_2\}$
T_1^2
$\{S_1\}$
$\{S_0\}$



$\{S_i\}$
$\hat{n}_i$
L_i
θ_i
$\{S_{i-1}\}$
$\hat{n}_{i-1}$
ϕ_i
$L_{4,i}$
$\{S_i\}$
$\hat{n}_i$
$L_{2,i}$
$L_{1,i}$
$\hat{n}_{i-1}$
$\{S_{i-1}\}$
$L_{3,i}$

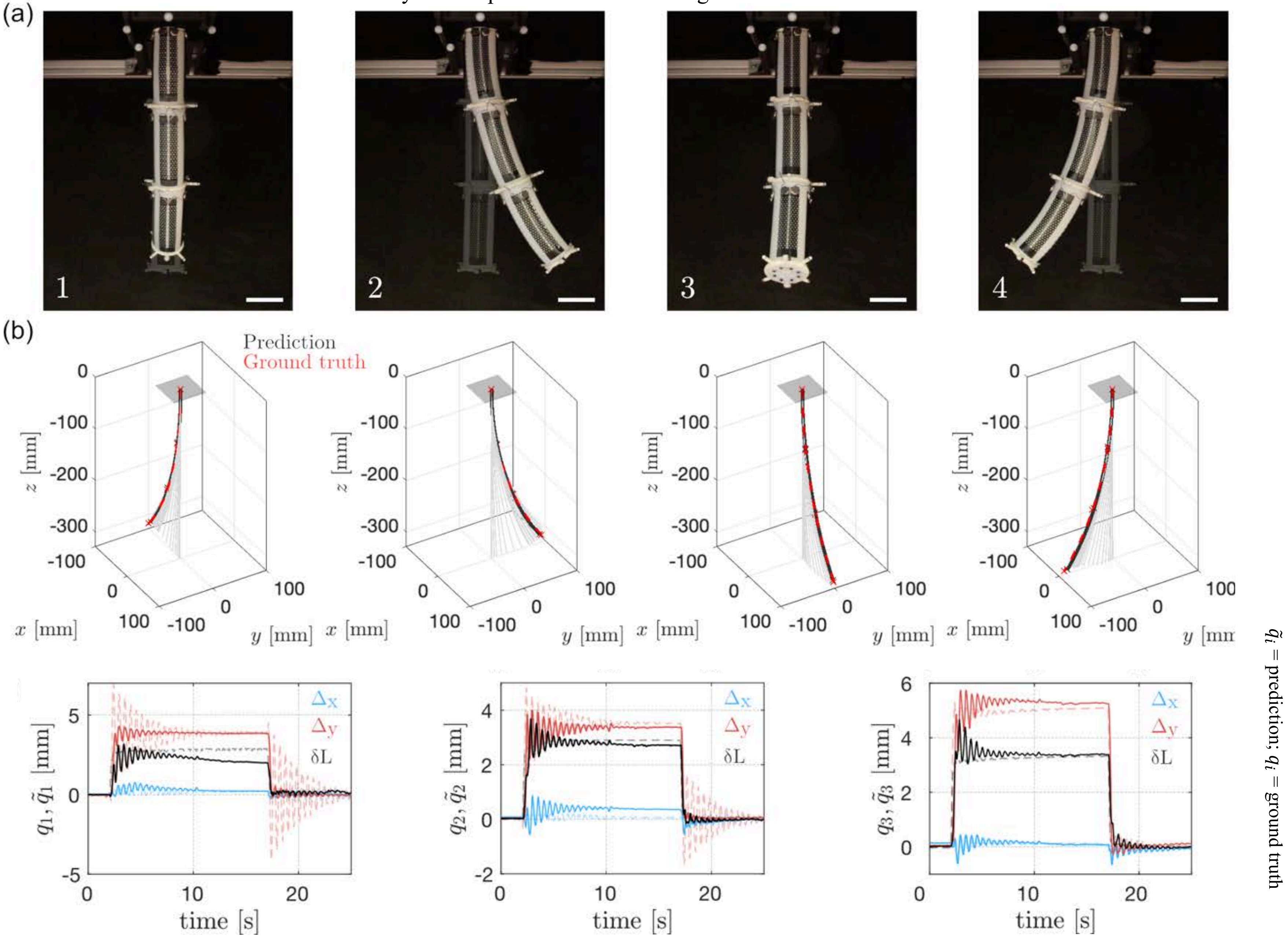

Training





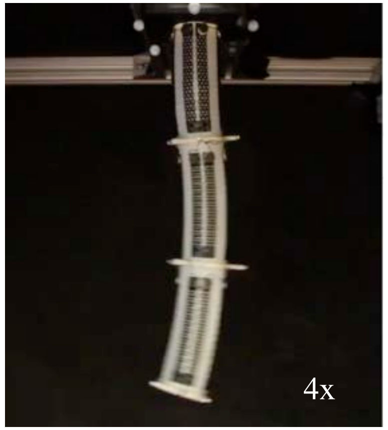
Prediction | Ground truth

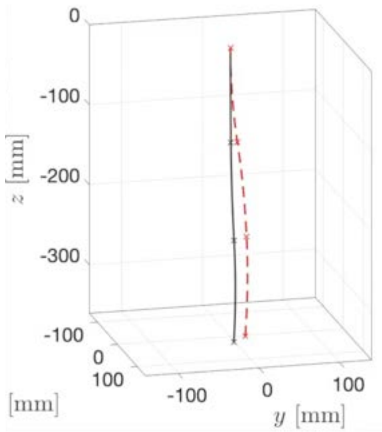
*Gray lines represent transient configurations



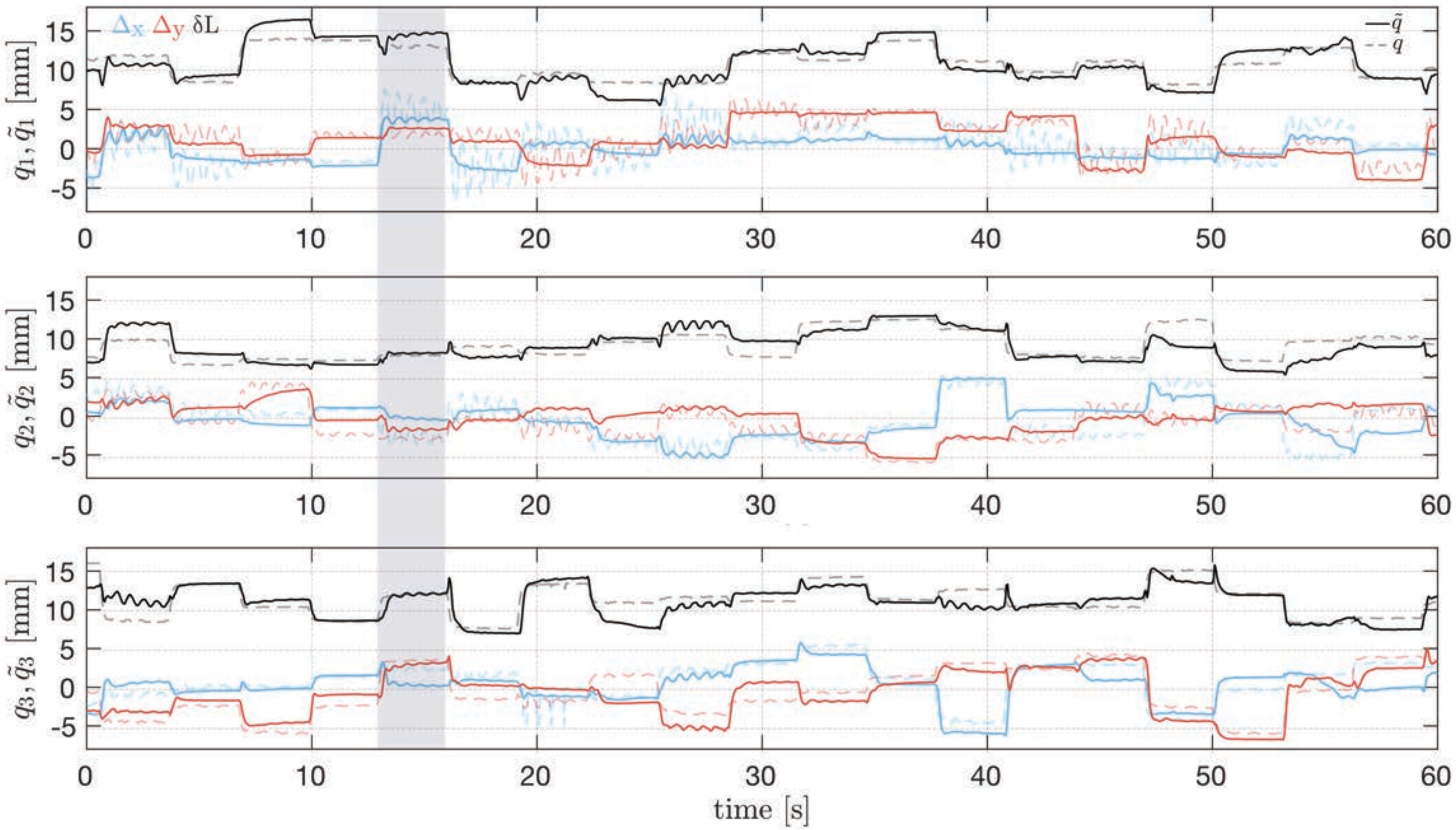
RL Truby*, C. Della Santina*, D. Rus. *IEEE Robotics and Automation Letters*, 2020. Accepted for IEEE ICRA 2020.

Random actuations





Prediction | Ground truth



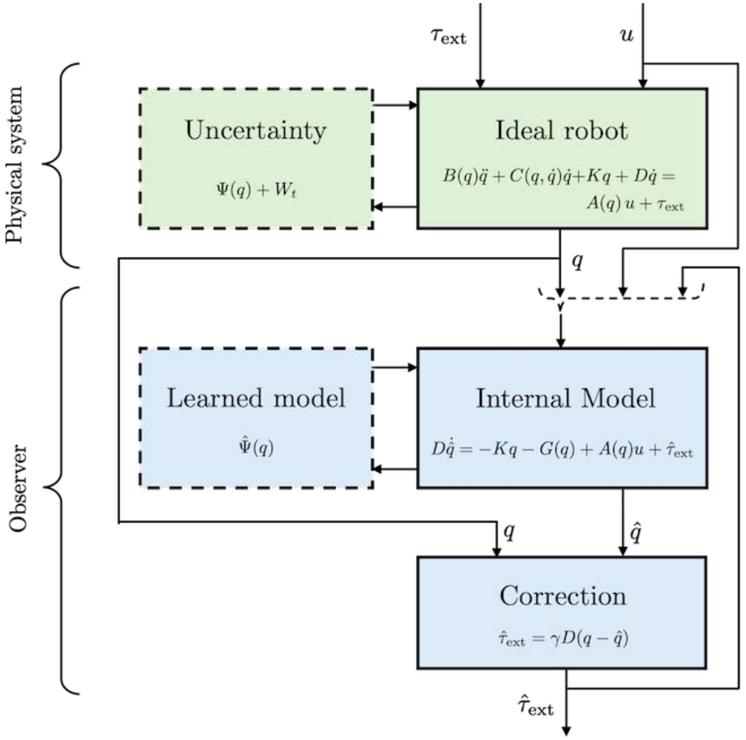
$\tilde{q}_i$ = prediction; q_i = ground truth

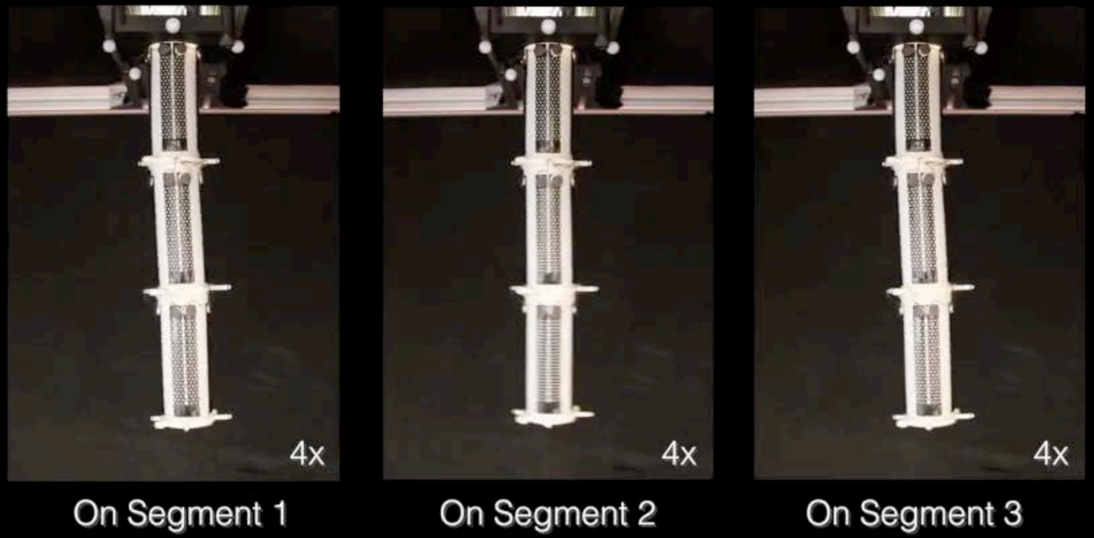
- Gray bar indicates one, 3-second random actuation interval
- Dynamic motions are filtered, especially for Δ_x and Δ_y
- Short time delay observed between $\tilde{q}_i$ and q_i
- **Dynamic behaviors are not fully captured**

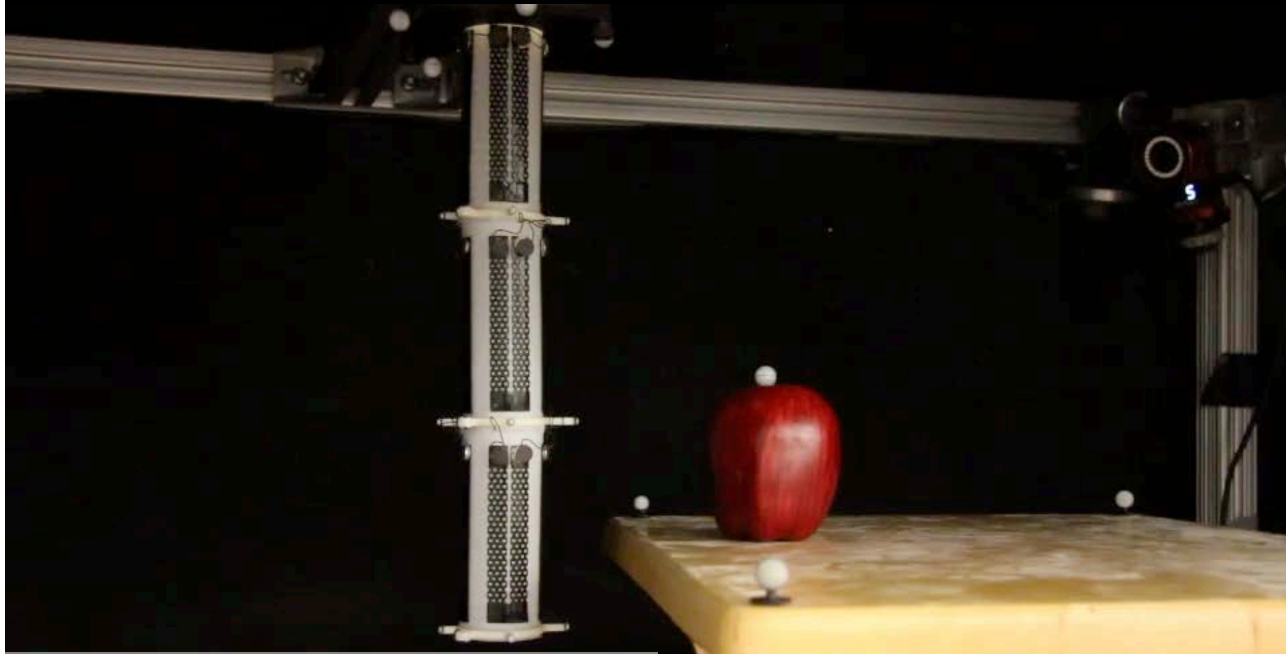
Predicted configuration approximates that at steady-state;
new sensors will improve predictions and feedback in control

RL Truby*, C. Della Santina*, D. Rus. *IEEE Robotics and Automation Letters*, 2020. Accepted for IEEE ICRA 2020.

Sensing Forces

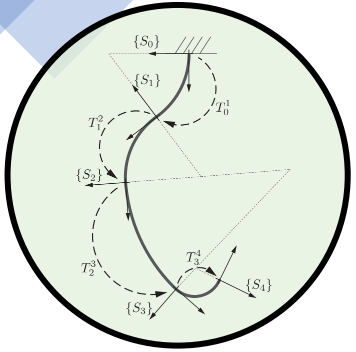




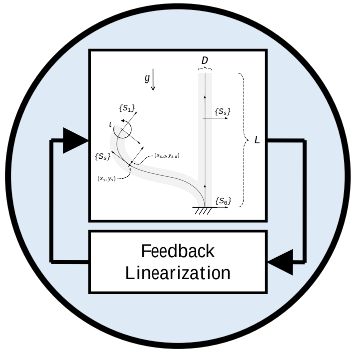


$$D\dot{\hat{q}} = -Kq - G(q) + A(q)u + \hat{\Psi}(q) + \hat{\tau}_{\text{ext}},$$
$$\hat{\tau}_{\text{ext}} = \gamma D(\hat{q} - q),$$

Della Santina C., R. L. Truby, D. Rus et al. "A New Class of Data-Driven Disturbance Observers for Estimating External Forces on Soft Robots." *RAL + IROS* 2020

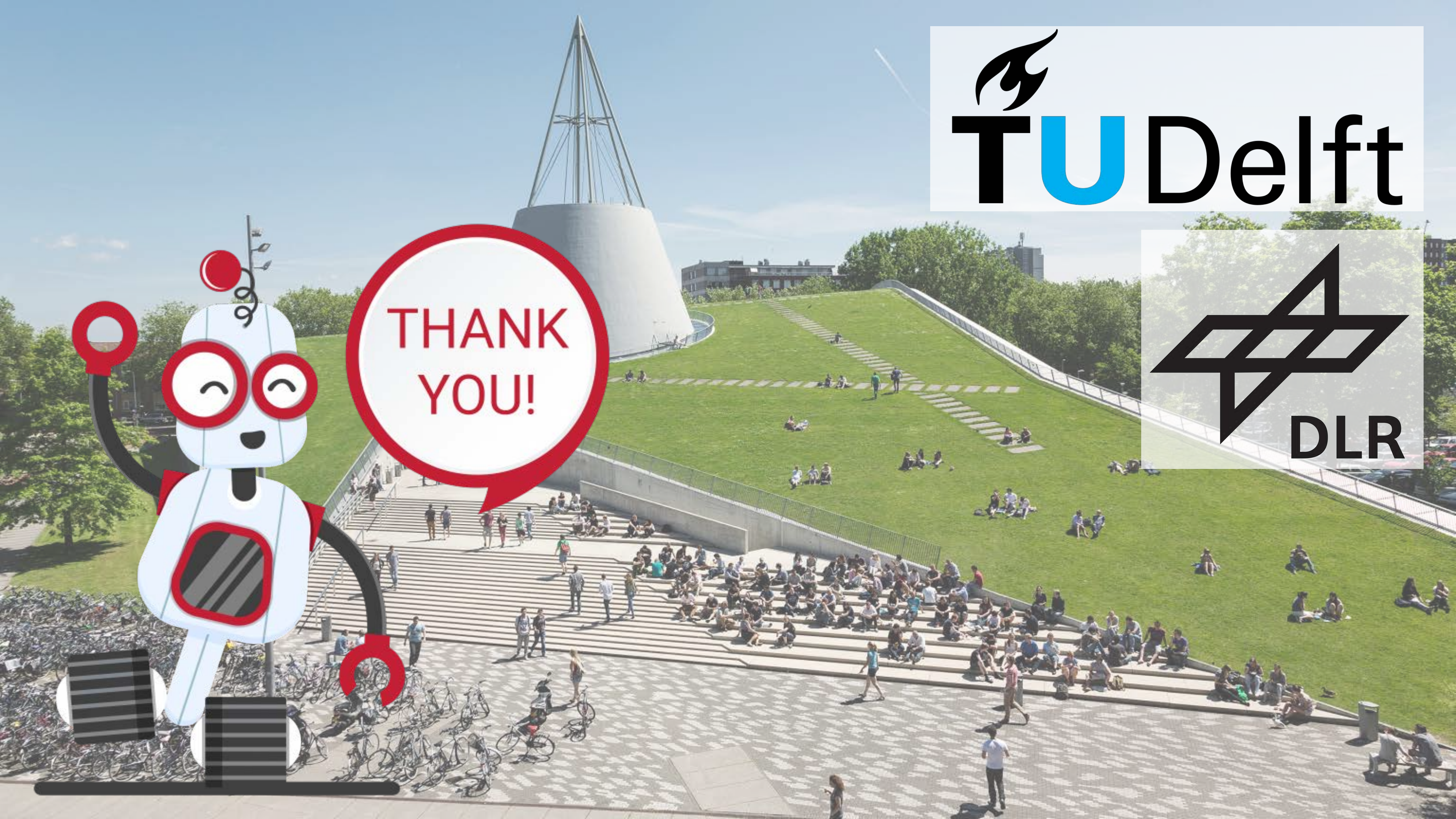


Feedback Model Based Control
Is Robust to Rough Approximations



If You Want to Dig More
Do That in a Control Oriented Way

If You Want to Stick to the Simple Model,
Consired Control-Driven Ways to Improve It


THANK YOU!
TU Delft
DLR